**ISC 5935 - Computational Tools for Finite Elements**

Homework#5

Assigned 1 October 2014, Due 8 October 2014

http://people.sc.fsu.edu/jburkardt/classes/fem 2014/homework5.pdf

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1. Download the Python code `bvp_01.py` from `http://people.sc.fsu.edu/~jburkardt/examples/fenics/bvp_01.py` and run it. This is to ensure FeniCS is installed and working on your machine. Save the output to file and turn it in.

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2. This question requires you to use FEniCS to solve problems that are variations of the problem in `bvp_01.py`. In each case write down the weak form and use piecewise linear basis functions on a mesh with 20 cells and plot the result using the built in `plot` command. Turn in the weak formulations as well as the plots.

a) Solve

$$\begin{aligned} -u''(x) + u(x) &= x,\ -2 \le x \le 1 \\ u(-2) &= 2 \\ u'(1) &= 5. \end{aligned}$$

b) Solve

$$\begin{aligned} -u''(x) + u(x) + 2u'(x) &= x,\ 0 \le x \le 1 \\ u(0) &= 0 \\ u'(1) &= 0. \end{aligned}$$

c) Solve

$$\begin{aligned} -\frac{d}{dx}((1+2x)\frac{d}{dx}u(x)) &= x,\ 0 \le x \le 1 \\ u(0) &= 2 \\ u'(1) &= 5. \end{aligned}$$

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3. For this question, we are not solving a PDE. Instead, we explore how well different finite element spaces, i.e. spaces of polynomial basis functions, approximate functions through interpolation. Write a new python script called `hw5q3.py` in which you define a two functions $f(x) = e^{-x^2}\sin(4\pi x)$ and $g(x) = \frac{|x-\pi/4|}{(x-\pi/4)}$. Use the 'Expression' statement. Note that `fabs` denotes the absolute value.

a) Use FEniCS to define a mesh on the unit interval, consisting of 5 cells. Now define function spaces `V1,V2` and `V3` on this mesh, consisting of piecewise linear-, quadratic-, and cubic polynomials respectively. Now form the interpolants `fi1,fi2,fi3` and `gi1,gi2,gi3` of $f$ and $g$ using basis functions in `V1,V2,` and `V3`. Use FEniCS to get the number of basis functions in each function space. Make a table of the number of basis functions in each space and turn it in.

b) Unfortunately FEniCs' built in plot function is limited, so we have to make use of pyplot in matplotlib ( `import matplotlib.pyplot as plt` ). Compute $f$ and $g$, as well as their interpolants, at 51 equally spaced points in $[0, 1]$ and make two plots, one for each function and its interpolants. Which function can be better approximated? Turn in the plots.

c) To assess the accuracy of an interpolant $\hat{f}$ of a function $f$, we need to compute an error norm $\|\hat{f} - f\|$. In this case we are interested in the maximum error norm, given by

$$\|\hat{f} - f\|_\infty = \max_{x\in[0,1]} |\hat{f}(x) - f(x)|$$

Compute an estimate of the maximum error for each of the six interpolants, based on the 51 points, to verify your findings. Turn in a table of the errors.

d) Now let's just use piecewise linear basis functions ( i.e. stick to `V1` ). Increase the number of cells from 5 to 10,20, and 40. In each case compute the maximum error based on 51 evenly spaced points. Turn in a table of the errors.