ISC 5935 - Computational Tools for Finite Elements
Homework #2
Assigned 10 September 2014, Due 17 September 2014
http://people.sc.fsu.edu/~jburkardt/classes/fem_2014/homework2.pdf

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1. Consider the following **classical** or **strong formulation** of a two-point boundary value problem (BVP):
Find a function u defined on [0,1] that is twice-continuously differentiable, such that:

$$\begin{aligned} -u'' + 7u =& x, 0 < x < 1, \\ u(0) =& 0, \\ u(1) =& 0. \end{aligned}$$

Having zero boundary conditions at both ends should make this problem simpler than the problem we looked at earlier in class.

- State the **weak** or **variational formulation** of the problem, supposing that V is the space of all continuously differentiable functions v defined over [0,1] with the property that $v(0) = v(1) = 0$. Your statement should begin:
Find a function u defined on [0,1] that is ??? such that':

$$\begin{aligned} ? =& ?, \\ ? =& ?, \\ ? =& ?. \end{aligned}$$

- State the **discretized weak** or **variational formulation** of the problem, assuming that V^h is an n-dimensional subspace of V with basis vectors $\psi_1(x), \psi_2(x), ..., \psi_n(x)$. Your statement should begin:
Find a function u^h defined on [0,1] that is ??? such that:

$$\begin{aligned} ? =& ?, \\ ? =& ?, \\ ? =& ?. \end{aligned}$$

Turn in your statements of the two formulations.

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2. Get a copy of the python program **fem1d.py** from
http://people.sc.fsu.edu/~jburkardt/py_src/fem1d/fem1d.html
and run it.

Alternatively, you are welcome to write a corresponding program in a language of your choice.

Turn in the printed output from your program, which should include the solution, exact solution, and error at each node.

3. The method of manufactured solutions is a way to go through the motions of solving a problem for which you have already cooked up the answer. Consider the following problem: *Find a function u defined on [0,1] that is twice-continuously differentiable, such that:*

$$
\begin{aligned}
-u'' &= ?, 0 < x < 1,\\
u(0) &= ?,\\
u(1) &= ?.
\end{aligned}
$$

and assume that we want the exact solution to be $u(x) = \cos(4 * \pi * x)$. Make a new copy of fem1d.py to solve this problem.

- Determine the right hand side and boundary conditions that must be specified in order that $u(x) = \cos(4 * \pi * x)$ will be the solution. Restate the problem to be solved by giving the explicit formulas for the "question mark" quantities.
- Modify the program so that it will try to solve your new problem.
- Run the program using N = 5, that is, with 5 elements (and 6 nodes.) The plot probably does not look great, and the maximum error will be relatively large.
- Now run the program using 5, 10, 20 and 40 elements. Make a table of the maximum error that occurred with each run.

Turn in your revised problem statement, a printout of your revised program, and the table of the results of the four program runs.