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A Review of Variational and Optimization Methods in Meteorology

by

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ABSTRACT

A condensed overview of variational and optimization methods in Meteorology is presented. It is aimed at giving the reader a short concise perspective of the developments in the discipline in the last thirty years and to present briefly recent developments in the application of optimization and optimal control theory in Meteorology.

INTRODUCTION

The first applications of variational methods in meteorology have been pioneered by Sasaki (1955, 1958) when he developed an initialization method based on the calculus of variations. In this general variational formalism one defines a functional - whose extremal solution minimizes the variance of the difference between observed and analyzed variable values, in a least-squares sense - subject to a set of constraints which are satisfied exactly or approximately by the analyzed values.

A variational functional is formulated, the minimization of which gives rise to a set of Euler-Lagrange equations, which are then solved numerically. A brief review of applications of the variational method in meteorology will be presented followed by a survey of the introduction of non-linear programming and optimization methods in meteorology and finally the introduction of optimal control theory methods (the adjoint model technique) in meteorology will close the review.

VARIATIONAL METHODS

In the application of variational methods a differentiation is made between a strong constraint (i.e. that an equality constraint should be identically equal to zero) and a "weak-constraint" where the equality constraint should only approximately be equal to zero. This method is equivalent to the first step in a penalty method. Stephens (1965) applied the variational method to analysis problems, using functionals formulated with weak constraints - and used weights which in a sense determine a low-pass filter.

In a series of papers Sasaki (1969, 1970a, 1970b) generalized his method to include time-variations and dynamical equations in order to filter high-frequency noise - and to obtain dynamically acceptable initial values in data void areas.

A multitude of papers applying these ideas appeared in the 1970's using the variational method with different constraint such as including the balance equation Stephens (1970), Barker et al. (1977).

Variational synoptic-scale analysis was carried out by Lewis (1972) and Lewis and Grayson (1972). Sheets (1973) applied the variational method to hurricane analysis, while Lewis and Bloom (1978) and Bloom (1983) used a variational adjustment using dynamic constraints in the analysis of mesoscale rawinsonde data. (See also Thompson (1969), Ray et al. (1980) and Testud et al. (1983) applied the method for computing velocity fields from Doppler Radar data. J. J. O'Brien (1970) used a variational formulation to obtain realistic estimates of the vertical velocity.

Middle and large-scale variational adjustment of atmospheric fields was performed by Stephens and Johnson (1978). A variational analysis method was carried out, applied towards the removal of Seasat Satellite Scatterometer winds by Hoffman (1982, 1984) using conjugate-gradient methods for the unconstrained minimization.

The use of inequality constraints in variational adjustment was introduced by Sasaki and McGinley (1982). Sasaki and Goerss (1982) used the variational approach for Satellite data assimilation.

A generalized variational objective analysis based on a duality between optimum interpolation and variational analysis and using a generalized cross validation was developed by Wahba and Wandelberger (1980), and Wahba (1981, 1982).

Seaman et al. (1977) used a variational blending technique over a large area based on a method of fields by information blending.

USE OF VARIATIONAL METHODS TO ENFORCE 'A POSTERIORI' CONSERVATION OF INTEGRAL INVARIANTS

Sasaki (1975, 1976) proposed a variational approach for enforcing 'a posteriori' constraints of mass and total energy conservation when solving the shallow-water equations. Bayliss and Isaacson (1975) and Isaacson (1977) independently proposed to linearize the conservative constraints about the predicted values by a gradient method also with the view of enforcing integral invariants conservation. The two approaches have been tested and compared by Navon (1981). Sasaki and Reddy (1980) used a similar method for enforcing potential enstrophy conservation.

VARIATIONAL NORMAL MODE INITIALIZATION AND RELATED ISSUES

Variational normal mode initialization was pioneered by Daley (1978), who combined the Machenhauer (1977) non-linear normal-mode initialization (NMI) with the variational procedure of Sasaki (1958) allowing the adjustment of the wind field to the mass-field or vice-versa based on presumed accuracy of observations (confidence weights).

Use of this concept was made by Daley and Puri (1980) for four-dimensional data-assimilation. Phillips (1981) proposed a slow-mode multivariate optimum interpolation and discussed the usefulness of a variational analysis.

Tribbia (1982) generalized variational NMI and used directly normal modes and longitude/latitude variable weights. His approach requires solution of a linear least-squares problem.

Puri (1982, 1983) used variational NMI to minimize the loss of surface pressure information. Temperton (1982, 1984) generalized the Tribbia (1982) approach for the variational NMI of the ECMWF gridpoint model. A variational bounded derivative method using a variational approach as well as a duality Augmented Lagrangian method is now being tested by Navon and Semazzi (1986). Le-Dimet et al. (1982) also proposed a variational approach to the problem of initialization.

Introduction of non-linear programming and optimization methods in Meteorology

The Augmented Lagrangian combined multiplier penalty method was proposed by Navon (1982a, 1982b) and implemented in different models by Navon and de Villiers (1983, 1986). Le-Dimet (1982a, 1982b), Le-Dimet and Talagrand (1985) and Le-Dimet and Segot (1985) also employed the Augmented-Lagrangian method. A constraint restoration method due to A. Miele et al. (1969) was implemented by Navon and de Villiers (1985).

Optimal control methods (the adjoint method)

The adjoint method is an application of optimal control theory where a functional is minimized by finding its gradient with respect to one of the analysis states (e.g. the initial state) and then using unconstrained minimization methods such as the conjugate gradient to iterate towards the optimal state. Finding the gradient involves use of adjoint equations. The method was pioneered by Marchuk (1974, 1982) and described by Kontarev (1980). Hall and Cacuci (1984) used the method to study sensitivity of numerical models with respect to physical parameters. Recent advances on this topic were done by Talagrand (1985), Le-Dimet and Talagrand (1985), Courtier (1985), Lewis and Derber (1985) and Derber (1985). Lewis and Derber (1985) used the adjoint method to solve a variational adjustment problem with advective constraints while Derber (1985) used the adjoint method for a variational 4-D data assimilation using quasi-geostrophic models as constraints. Courtier (1985) applied it to a shallow water equations model. Le-Dimet and Talagrand (1985) used the method for data assimilation with a 1-D shallow water equations model.

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