Objective Analysis of Pseudostress over the Indian Ocean Using a Direct-Minimization Approach

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ABSTRACT

A variational approach is used to develop an objective analysis technique which produces monthly average 1-deg pseudostress vector fields over the Indian Ocean. A cost functional is constructed which consists of five terms, each expressing a lack of fit to prescribed conditions. The first expresses the proximity to the input (first-guess) field. The second deals with the closeness of fit to the climatological value for that month. The third is a measure of data roughness, and the fourth and fifth are kinematic constraints on agreement of the curl and divergence of the results to the curl and divergence of the climatology. Each term also has a coefficient (weight) which determines how closely the minimization fits each lack of fit. These weights are determined by comparing the results using various weight combinations to an independent subjective analysis of the same dataset. The cost functional is minimized using the conjugate-gradient method.

Results from various weight combinations are presented for the months of January and July 1984 and the results examined in terms of these selections. Quantitative and qualitative comparisons to the subjective analysis are made to find which weight combination provides the best results. It was found that the weight on the second term balances the influence of the original (first-guess) field and climatology. The smoothing term weight determines how wide an area deviations of the first guess from climatology is affected. The weights on the kinematic terms are fine-tuning parameters.

1. Introduction

Nowhere else in the world do the monsoon winds change like those in the northwest region of the Indian Ocean basin. The semiannual reversal of winds—northeasterly in the northern winter and southwesterly in the northern summer—is unique and results in interesting phenomena being generated in the upper layers of the ocean. For example, the Somali current which reverses direction according to the season has a dynamically active circulation structure characterized by a complex set of eddies and fronts. Luther and O'Brien (1985) and Luther et al. (1985), used a reduced-gravity, nonlinear ocean model forced by climatological and FGGE winds, to simulate successfully many of the features of the Somali current system. They found that the wind stress curl and the strong gradient in the curl associated with the Findlater (1971) jet (a southerly jet-like wind structure appearing in the northern summer) greatly influenced the development of the Somali current system.

Several studies (Bruce 1983; Dutch Atlas 1952; Hantel 1971; Hastenrath and Lamb 1979; Hellerman and Rosenstein 1983; Fu et al. 1983, to mention just a few) have developed and/or looked at the climatological values of winds over the Arabian Sea or Indian Ocean. Other studies (Cadet and Desbois 1981; Findlater 1971; Schott and Fernandez-Partagas 1981; Wylie and Hinton 1982) have focused on particular years or seasons, especially the summer of 1979. Few studies (Cadet and Diehl 1984) have focused on providing or examining fine resolution regularly gridded data for studying the interannual variability of large-scale winds or for use in forcing ocean models of the region. (There are, however, some operational products of unknown quality.) Interannual variability of the winds in this region is important and has been studied in connection with other wind systems as well (Barnett 1983).

In this paper, we will explain a novel method used to produce monthly average pseudostress values on a

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1-deg mesh over the entire Indian basin, 30°S to 28°N, using kinematic in place of dynamic constraints. These values were calculated using a variational technique involving direct minimization of a cost functional consisting of a number of terms each expressing a lack of fit to a prescribed condition. Associated with each term in the cost functional is a weight which will be determined by comparing the results of various weight combinations to an independent analysis of the same dataset (see section 3). This variational method is a nonlinear least-squares problem requiring an iterative method of solution; the first guess being the monthly available surface marine reports which were interpolated (where necessary) to fill all grid points. The results have been used in forcing the model of Luther et al. (1985) for the years 1977–85. Results from this ocean model from the fall of 1985 have been validated by comparing them to collocated U.S. Navy bathythermograph and NOAA satellite data (Simmons et al. 1988).

The paper will follow this guideline: Section 2 will discuss the raw data, the direct minimization technique and the implementation of the technique. In section 3, we will examine a couple of months in 1984 to determine the best selection of weights for each of the terms in the cost functional. Then, in section 4, we will present some conclusions, and summarize in section 5.

2. Minimization method

a. Data filtering

The raw data for this analysis was obtained from the National Climatic Data Center, Asheville, North Carolina. Marine surface observations from the TD-1129 tapes were obtained for the year 1984, and wind observation data were extracted over the Indian Ocean region, 30°E to 122°E, 26°S to 26°N. For each month in this time period, there were approximately 10 000–20 000 such observations in this area. All winds with a reported speed greater than 40 m s^{-1} were removed (as they were considered nonphysical, hence erroneous). Observations were then converted to pseudostress as defined by

$$|\mathbf{V}| = (u^2 + v^2)^{1/2} \tag{1}$$

$$\tau_x = u \cdot |\mathbf{V}|$$

$$\tau_y = v \cdot |\mathbf{V}|, \tag{2}$$

where u and v are the eastward and northward components of the reported wind. From pseudostress, one could easily compute wind stress using a prescription for a drag coefficient.

Objective filtering was applied to the extracted data to aid in eliminating questionable reports. The filtering considered the two components τ_x and τ_y independently. In each month's data, a basin-wide average was found for both τ_x and τ_y. Then, upper and lower limits of each component were set to be this average value plus (minus) 30 times this average. If either component of an individual wind report fell outside the range, the entire wind observation was deleted. Typically, less than 1% of all the reports were deleted in this fashion.

The remaining reports were then averaged in 1° lat by 1° long boxes. The final filtering dealt with each box individually. For each box with greater than five observations in the month, reports were removed with either component (τ_x or τ_y) outside the range −3.7 to 3.7 standard deviations. Again, typically less than 1% of the reports were removed.

The result is a monthly average pseudostress vector map on a 1° × 1° mesh with empty boxes being found primarily in areas located outside the shipping lanes. These locations were filled using bilinear interpolation, and grid points over land were assigned values by extrapolating the nearest ocean boxes.

b. Minimization technique and the cost functional

The first step in implementing direct minimization is designing the cost function which will be minimized. It will be a measure of lack of fit of the data according to certain prescribed conditions which may be dynamically or statistically motivated. We know from climatology, for example, what characteristics the surface wind pattern should generally have. The wind pattern should also be "smooth" to some extent. Thus, some measure of roughness and some measure of lack of fit to climatology should be included in the cost function.

In this study, the only information available will be (a) ship report averages on a 1° mesh, and (b) a 60-yr pseudostress climatology based on Hellerman and Rosenstein (1983). This climatology consists of 12 average months of pseudostress on a 1° mesh. To calculate this pseudostress from the original 2-deg Hellerman and Rosenstein wind stress values, the overall average Hellerman and Rosenstein wind stress map over the Indian Ocean region was compared to a much noisier 60-yr pseudostress climatology average computed from the National Climate Data Center (NCDC) Global Marine Sums. The factor (equivalent of the drag coefficient 2.7 × 10^{-3}) was taken out of each of the 12 wind stress maps to result in a 12 month pseudostress climatology. A 1° mesh was then calculated using bilinear interpolation. The pseudostress climatology from the NCDC was not used because it contained data-void regions and was very noisy.

The key ingredients in our objective scheme are the inclusion of two kinematic constraints similar to those used by Hoffman (1984). Since it is not practical to use a dynamic model of the atmospheric boundary layer over the sea, we choose to require the analysis to be similar to the curl and divergence of the climatology as well as the climatology itself.

The cost functional, F, which is used to find objectively derived monthly maps of pseudostress is defined as follows:

$$F = \frac{\rho}{L^2} \sum_x \sum_y [(\tau_x - \tau_{x_0})^2 + (\tau_y - \tau_{y_0})^2]$$
$$+ \frac{\gamma}{L^2} \sum_x \sum_y [(\tau_x - \tau_{x_c})^2 + (\tau_y - \tau_{y_c})^2]$$
$$+ L^2\lambda \sum_x \sum_y [(\nabla^2(\tau_x - \tau_{x_c}))^2] + (\nabla^2(\tau_y - \tau_{y_c}))^2]$$
$$+ \beta \sum_x \sum_y [\nabla \cdot (\tau - \tau_c)]^2$$
$$+ \alpha \sum_x \sum_y [\hat{\mathbf{k}} \cdot \nabla \times (\tau - \tau_c)]^2 \quad (3)$$

where τ_x, τ_y are the eastward and northward pseudostress components to be determined; τ_{x_0}, τ_{y_0} are the components of the gridded marine reports; τ_{x_c}, τ_{y_c} are the components of the pseudostress climatology; τ, τ_c are the determined and climatology pseudostress vectors; and L is a length scale (chosen to be 1-deg latitude) which makes all terms dimensionally uniform and same order of magnitude. The coefficients (actually weights) ρ, γ, λ, β, and α control how closely the direct minimization fits each constraint (lack of fit). They represent the relative contribution of each of the five terms to the cost functional, F. The kinematic terms were approximated by second-order finite differencing in cartesian coordinates.

The cost functional has five terms. The first term expresses the proximity to the original (input) data. (This weight is always 1.) The second term deals with the closeness of fit to the climatological value for that month. A higher value leads to a closer likeness to the climatological value. The third term is a measure of the data roughness, and controls the "radius of influence" of an anomaly in the input winds or in the climatological values. It can be called a "smoothing term" or a "penalty function."

The last two terms are the boundary layer kinematic terms. They force the results to be comparable to the climatology, but not in the direct sense. They control the degree to which the divergence and curl of the resulting vector field approximate the kinematics of the climatology. The five terms of the cost functional address some of the possible constraints. Other possibilities include time evolution constraints, kinetic energy constraints, pressure gradient terms, etc.

The means of selecting the proper weights has not yet been addressed. This problem of defining "proper" values for the five weights is the key to the analysis. From results in this study, small variations in the weights for the derivative terms (smoothness, divergence, and curl) had little effect on the results. The second weight was critical for it balanced the overall magnitude of the results between the climatological norms and the ship reports (usually larger magnitude). The weights can be thought of as empirically determined tuning parameters. These weights could be chosen by objective means: in theory the method of generalized cross validation (Wahba & Wendelberger 1980) could be used but the computation would be impractical with this size dataset (solve a set of 6266 linear equations). In addition, cross-validation requires "valid" data, something which is very difficult to assess. The adjoint model technique (Cacuci 1981; Hall and Cacuci 1983; Talagrand 1985) can aid in a sensitivity study of the critical tuning parameters. None of these objective methods could tell us what the "correct" values of the weights should be since the "correct" solution is not known. Instead, comparisons to independent analyses will be used.

Only the 3133 points over the ocean were varied, and since a τ_x and τ_y must be varied at each point, the cost functional included a total of 6266 variables. This functional, when minimized by varying the resulting data, will then be a best fit to this prescribed function according to the selected weights. Since one of the weights is arbitrary, in this study we set ρ to unity. The weights could be functions of location (regional effects), or other such dependencies; however for the present study, they were chosen to be constant throughout the domain.

c. *The conjugate-gradient technique for unconstrained minimization*

From the different techniques for minimizing a general nonlinear function of n variables, the conjugate-gradient method is the fastest and the only one practically implementable where n is large (i.e., $n \approx 10^4$) (See Navon and Legler 1987.) Methods with better convergence rates than the conjugate-gradient, like the Newton and quasi-Newton methods, require storage of Hessian matrices of size ($n \times n$) and are out of consideration for the purpose of our study. Conjugate-gradient algorithms require storage of only a few vectors of length n, typically between six and eight n-vectors of storage (Powell 1977; Shanno 1978a; Shanno and Phua 1976, 1980). The cost function F to be minimized is a discrete expression including double summation on the different measures of lack of fit as well as horizontal finite difference operators.

For the conjugate-gradient method, the gradient, g, of the objective cost function F must be calculated where

$$g = \nabla F = \left(\frac{\partial F}{\partial \tau_{x_{11}}}, \frac{\partial F}{\partial \tau_{y_{11}}}, \cdots \frac{\partial F}{\partial \tau_{x_{NxNy}}}, \frac{\partial F}{\partial \tau_{y_{NxNy}}} \right) \quad (4)$$

where Nx and Ny are number of grid points in longitude and latitude respectively; $n = 2NxNy$, i.e., n is twice the number of grid points in the analysis domain. In our case we have $n = 6266$. The calculation of the gradient is done from the discretized version of F and it follows the approach of Navon (1981), and Hoffman (1984) by treating only the few adjacent grid points for which $\partial F/\partial \tau_{x_{ij}}$ or $\partial F/\partial \tau_{y_{ij}}$ are different from zero.

The minimization code used here is the Shanno-Phua (1980) code CONMIN which was found to be the most efficient of the available conjugate-gradient codes in tests conducted by Navon and Legler (1987). The Shanno-Phua method is a limited memory quasi-Newton method which updates the Hessian matrix with a limited number of BFGS (Broyden–Fletcher–Goldfarb–Shanno) updates, but never stores the $(n \times n)$ Hessian matrix, and the only additional storage are the vectors defining the updates.

Details of the algorithmic implementation of the conjugate-gradient method CONMIN will be given in the Appendix. An accuracy criterion of

$$\|g(\mathbf{x})\|^2\epsilon_a \qquad (5)$$

was used for stopping the iteration.

In our case we used $\epsilon_a = 10^{-2}$. A total of nine iterations and eight Beale restarts was required. Part of the success of the conjugate-gradient method is to be attributed to the nondimensional scaling of the cost functional F, as well as the fact that for the problem at hand, the functional F is quadratic and convex. In particular, the smoothing function (multiplied by λ) adds considerable convexity to the functional. As such, the conjugate-gradient method is well conditioned and moreover we seem to be able to calculate a unique minimum.

3. Results

Since the weights in the functional can be tuned to provide different results corresponding to various alterations in the weights, the results should be compared to an already known field or other verifying data. Thus, for this study, the results from the minimization method were compared to an independent subjective analysis of the same surface marine reports. For this subjective analysis, the marine observations for 1984 used as input into the objective method were first binned into 2° lat by 5° long quadrangles. In each quadrangle, the wind reports were filtered by removing the highest and lowest 10% of the observed components. In addition, based on the new averages, a three-standard deviation test was used. The resulting values were hand analyzed by an experienced meteorologist to remove or change any remaining questionable values. The resulting fields were digitized onto a 2° × 2° grid. This analysis scheme is the same as the one used to produce 2° grid pseudostress values in the tropical Pacific (Goldenberg and O'Brien 1981; Legler and O'Brien 1985). This subjective analysis was completed before the objective method was implemented and, thus is a totally independent analysis of the same data.

Since the objective results are on a 1-deg grid, and the subjective analyses are on a 2-deg grid, the objective results were averaged in 2-deg boxes. This is a disadvantage because selecting the weights by comparison of the 2-deg results will neglect how the smaller scales may influence the weight selection. If the objective results match the subjective results exactly, then the methods might be considered identical; however, the goal is not to duplicate the subjective method objectively. Rather, it is to select the corresponding weights of the cost functional to give realistic results. (The subjective analyses are considered realistic.) As will be shown, the weight selection which offers the best quantitative comparison to the subjective analyses is not always the best selection.

It is implicitly understood that the weights would be a function of the individual month if exact matches were the desired result. This would force the selection of the weights to be an objective process. Instead, the focus of the weight selection will be on the results for a pair of months which were chosen to represent the extremes of the monsoon cycle.

a. July 1984 study

The subjective analysis for July 1984 (Fig. 1) is different in many respects from climatology. The southeast trades are strong. Winds southeast of Madagascar are more southerly than climatological winds. The area of weakest winds, at 65°–70°E along the equator, agrees fairly well with climatology. The maximum wind velocity of the Findlater Jet is less than the climatological mean. Winds in the Bay of Bengal are stronger than expected with especially large westerly values stretching into the midbay region. The South China Sea (SCS) winds are normal. The curl of the subjective results is larger in magnitude but compares favorably to the climatology curl, except off the coast of Somalia where the curl is negative instead of the usual positive (upwelling) values.

For the first attempt in determining a good weight selection, all the weights were set to 1.0 except for γ which was set to 0.1 to force less dependence on climatology. The fields in Figs. 2 and 3 illustrate the typ-

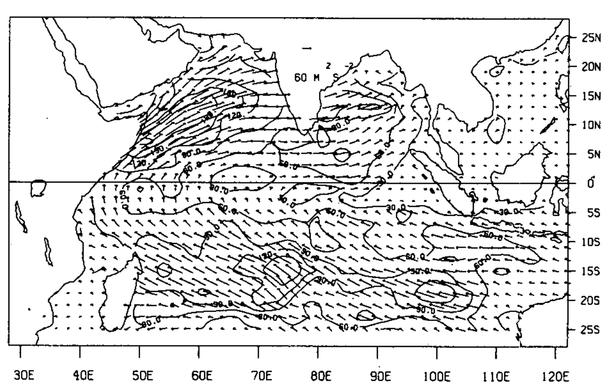

FIG. 1. Subjective analysis of pseudostress for July 1984. Contours are of pseudostress magnitude and the interval is 30 m² s⁻².

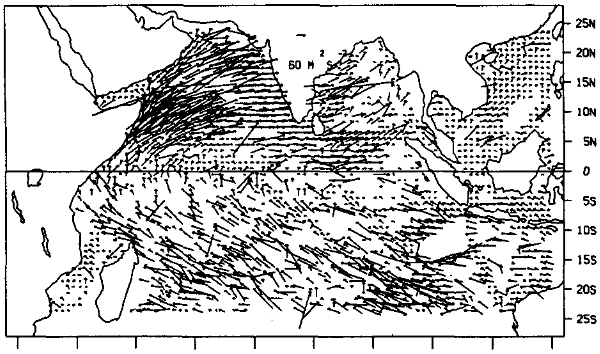

FIG. 2. Pseudostress data from ship reports, filtered and binned onto 1° boxes for July 1984.

ical progression of the variation technique. Figure 2 shows the binned pseudostress data for July 1984. In Fig. 3, a sample box is shown in three stages of the technique and then a comparison of the resulting field to the first guess. The resulting map (Fig. 4) turns out well; however, it becomes immediately obvious that this result was not satisfactory because of (i) presence of errors at the eastern and southern boundary regions, (ii) large comparative differences with the subjective analysis at critical areas (Findlater jet and Bay of Bengal), and (iii) a general need for smoothing.

In order to help understand the influence of each of the terms of the cost functional, each weight was varied to 0 then to 5 while the others remained constant. A short description of each resulting map will be presented. In addition, to quantify how each of the results compares to the target (subjective) analysis, a group of statistics will be presented in Table 1 on the differences (objective − subjective). We shall call the first statistic the average vector difference. It is the magnitude of the vector differences (objective result − subjective result), averaged over the basin. The second statistic we shall call the energy of the difference field. It is the sum of the squares of the difference field components (comparable to difference in the kinetic energy of the fields).

The first two cases deal with the γ parameter: how much climatology should be included in the result. For the case when $\gamma = 0$ (the direct difference to climatology not considered) (Fig. 5), the result is very noisy. Maxima and minima are more extreme and are resolved into smaller regions; e.g., examine the Findlater jet and southern trades.

In contrast, when $\gamma = 5.0$, the climatology becomes more influential—the overall average vector magnitude is smaller. The direction and magnitude of many of

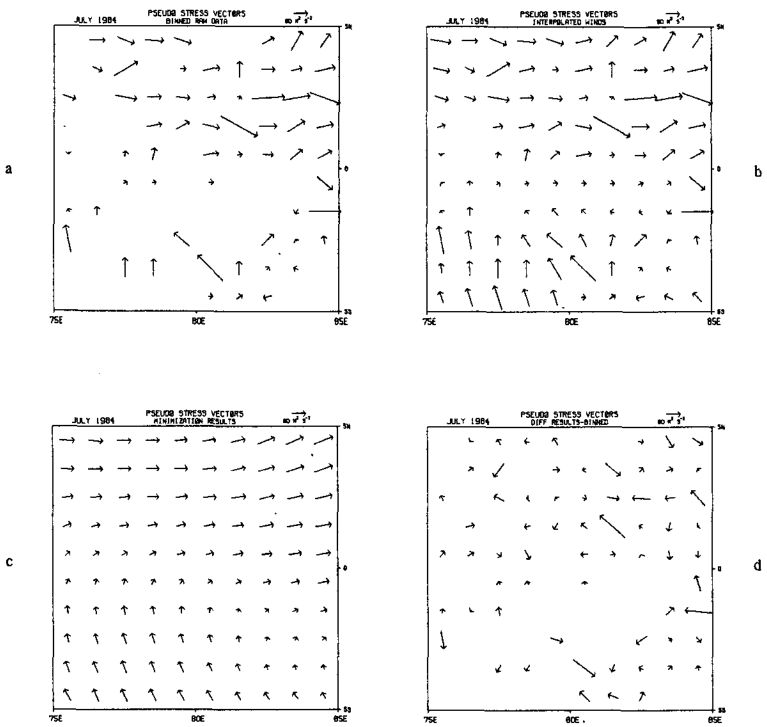

FIG. 3. Four stages of the data analysis in a sample region for July 1984. (a) Filtered and binned pseudostress values, (b) the data field after interpolation has been applied, (c) results of the minimization, and (d) vector difference (minimization results − binned data field).

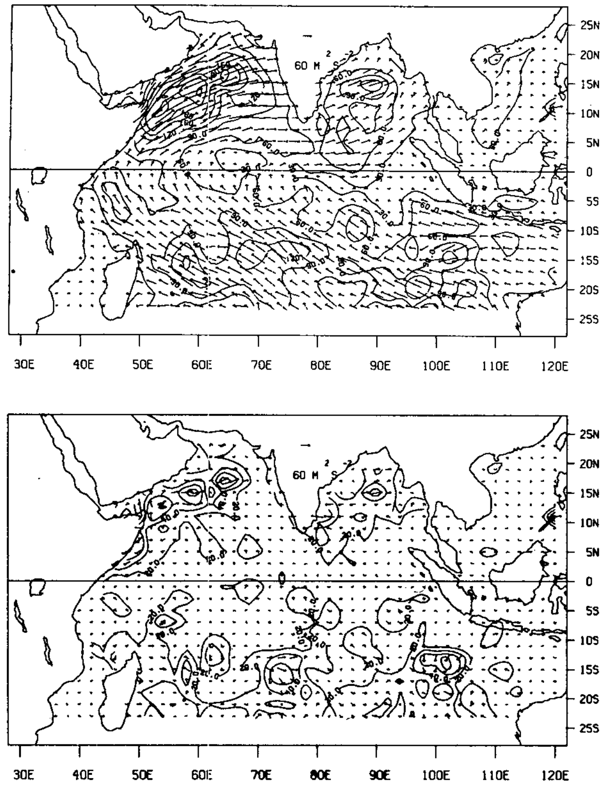

FIG. 4. (a) Pseudostress vector results of the variational (objective) analysis binned onto a 2° grid for July 1984. Contour interval is 30 $m^2 s^{-2}$. (b) Vector difference (objective results − subjective results). Contours are the vector difference magnitudes, and interval is 20 $m^2 s^{-2}$. This is the case for $\gamma = 0.1$, $\lambda = 1.0$, $\alpha = 1.0$, $\beta = 1.0$.

the maxima in the southern hemisphere are more tolerable. The energy of the difference field is smaller as is the average vector difference, but the maxima of the Findlater jet are less than the reported wind values. In the Bay of Bengal, large magnitude westerlies were greatly reduced in areal coverage and magnitude. In both areas, the curl does not correspond very well to either the curl climatology or the subjective analysis curl.

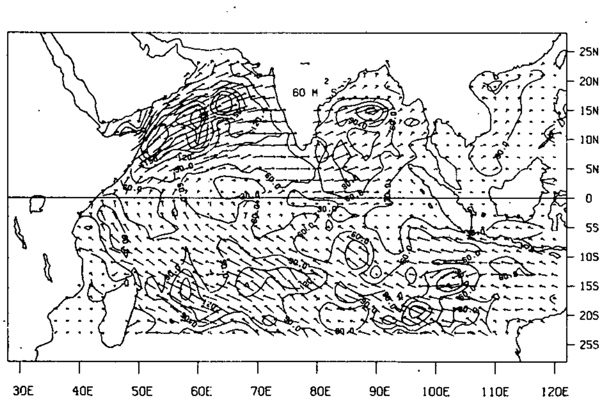

FIG. 5. Pseudostress vector results of the variational analysis binned onto 2° grid for July 1984. Contour interval is 30 $m^2 s^{-2}$. This is the case for $\gamma = 0$, $\lambda = 1.0$, $\alpha = 1.0$, $\beta = 1.0$.

The next two cases deal with the weight of the smoothing term, λ. Note this term forces the differences to be smoothed (spread) out over other grid points. Since the Laplacian operator is most effective on smaller scale wavelengths, the change in the value of λ from 0 to 5 (Fig. 6) is more evident on the 1-deg grid results.

This term controls the areal spreading of what balance exists between the original and climatological observation as defined by the first two terms. In both figures (6a and 6b), the original observations are very dominant because of the selection of γ ($\gamma = 0.1$). For $\lambda = 0$, the 1-deg field is very noisy; averaging the results to get the 2-deg map virtually erases this variability. For $\lambda = 5$, however, because of the increased weight on smoothing, the large differences to climatology have been distributed over several grid points in both x and y directions. Although the magnitude of this "noise" has decreased; the magnitude of surrounding points is increased due to the higher magnitude (as compared to $\lambda = 0$) differences being "smoothed" out. The kinetic energy of the difference field reflects this (Table 1).

The first three components of the cost functional do not deal directly with physical characteristics of the

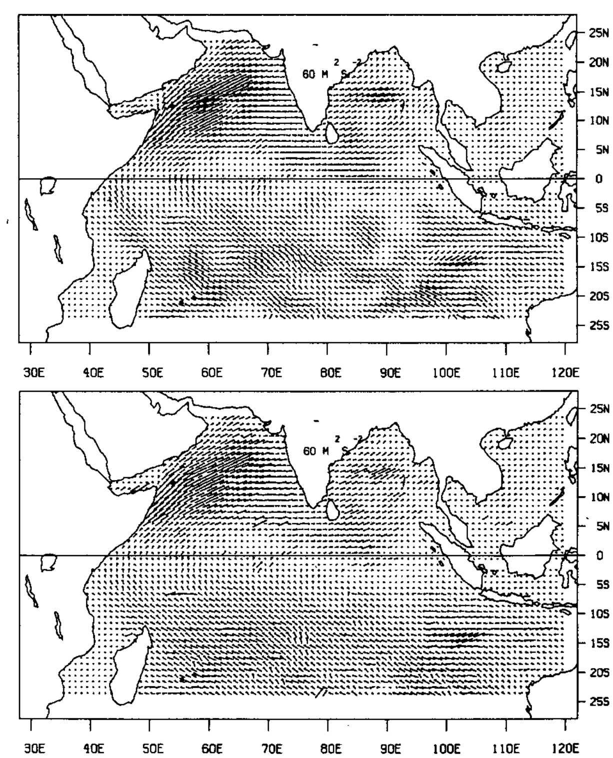

FIG. 6. Pseudostress vector results of the variational analysis on a 1° grid for July 1984. (a) This is the case for $\gamma = 0.1$, $\lambda = 5.0$, $\alpha = 1.0$, $\beta = 1.0$. (b) This is the case for $\gamma = 0.1$, $\lambda = 0.0$, $\alpha = 1.0$, $\beta = 1.0$.

TABLE 1. Error characteristics for the difference field (objective results minus the subjective results) for the July 1984 case.

Weights				$\lvert\overline{\mathbf{V}_D}\rvert$	Kinetic energy
γ	λ	α	β	($m^2 s^{-2}$)	($\times 10^4 m^4 s^{-4}$)
0.1	1	1	1	19.4	50.7
0	1	1	1	21.1	62.5
5	1	1	1	22.1	60.7
0.1	0	1	1	20.0	56.5
0.1	5	1	1	21.3	62.3
0.1	1	0	1	21.7	65.3
0.1	1	5	1	19.5	51.7
0.1	1	1	0	21.7	65.5
0.1	1	1	5	19.4	50.8
2.0	2	10	10	17.5	38.9
1.5	2	30	30	16.6	36.1

winds. That is not to say they do not produce results with realistic properties. They relate the results to two kinematic properties of the climatology: divergence and relative vorticity (curl). Although monthly variability is large about this climatology, the properties of this mean do reflect typical patterns. The next two cases deal with the first of these two kinematic constraints—the curl of the data.

Comparing the vector results when $\alpha = 0$ and when $\alpha = 5$ is difficult. The vector differences are very subtle and widespread. However, in the curl fields (Fig. 7), the differences are more obvious. Overall, the increased weight provides a smoother curl field removing some small spurious extrema. The weight of 5 did improve the analysis (by bringing the curl field more in agreement with climatology) according to quantitative results in Table 1. Note also that even though the vorticity map changed significantly, the divergence map of the same results showed no perceptible differences. This term is a fine tuning parameter for the vector field.

The last case in this study tests the weight of the divergence term. As in the previous discussion, increased β provides a better quantitative and qualitative comparison to the subjective analysis. The results when $\beta = 0$ are nearly identical to those when $\alpha = 0$ because the weight of 1 as opposed to 0 is insignificant. When $\beta = 5$, the divergence field is smoother than for $\beta = 0$, but the vorticity fields are very similar. Thus α and β require larger magnitude change to effect the vector results. They have a less subtle effect on the derivative fields.

Each term in the cost functional has a unique (filtering) effect on the results. It is a simple task to vary *one* weight at a time to determine the best value for comparative purposes, as addressed in subsection 3b below. It is much more difficult to find the proper combination of many weights.

b. Selection of weights

Again, we will not use an objective scheme to select the proper weights because there is no acceptable method of objectively determining them. However, we have made the premise that the independent subjective analyses of the same input data will be the target solution. Small randomly distributed differences between the objective results and the subjective analyses will be tolerable; coherent errors with large magnitude will indicate the need to alter the weights, provided these new weights do not jeopardize the results in other regions to a larger extent.

Since an independent increase (the kinematic weights more so: climatology less so) of each of the weights improved the results, the next case (Fig. 8) will have $\gamma = 2$, $\lambda = 2$, $\alpha = B = 10$. The kinetic energy of the difference field decreased dramatically. However, the Findlater jet is still too weak in a large area. In addition, the westerlies in the Bay of Bengal are weak and there are several areas of the southern trades where the winds are also too weak. This widely distributed weakness in the winds indicates the weight on climatology is too high; reducing it will allow more energy in the results since the original data is much more energetic than the climatology. The larger kinematic weights have also improved the analysis, as the curl and divergence fields are more similar to the subjective fields. Adjusting the γ weight, first to 1.5 and then to 3.0 with increased values of the kinematic weights re-

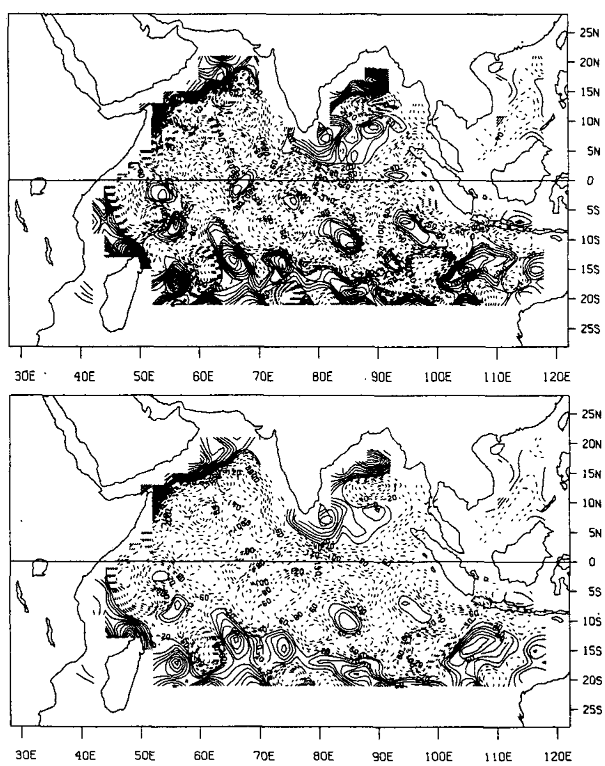

FIG. 7. Curl of the pseudostress for July 1984 objective results. The contour interval is 0.2E−04, and the labels are scaled by 0.1E+07. (Top) This is case for $\gamma = 0.1$, $\lambda = 1.0$, $\alpha = 0.0$, $\beta = 1.0$. (Bottom) This is case for $\gamma = 0.1$, $\lambda = 0.0$, $\alpha = 5.0$, $\beta = 1.0$.

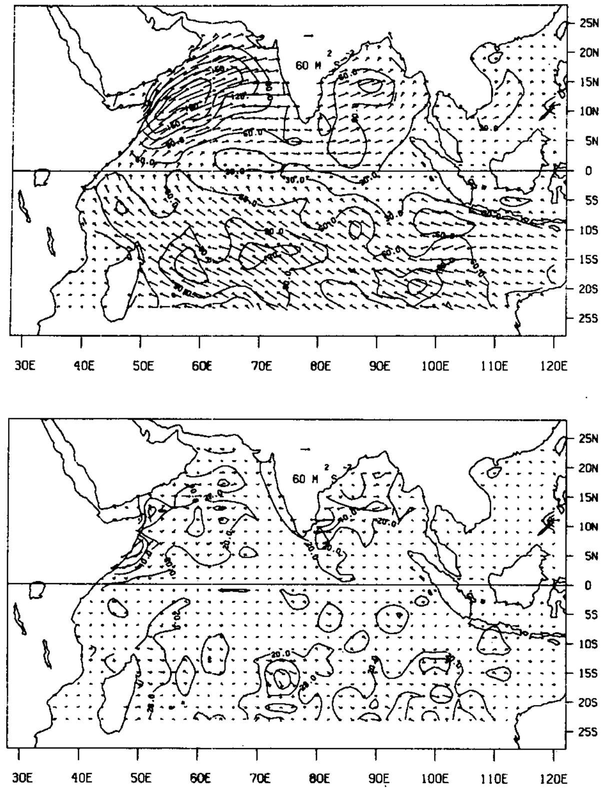

FIG. 8. (a) Pseudostress vector results of the variational (objective) analysis binned onto a 2° grid for July 1984. Contour interval is 30 $m^2 s^{-2}$. (b) Vector difference (objective results − subjective results). Contours are the vector difference magnitudes, and interval is 20 $m^2 s^{-2}$. This is the case for $\gamma = 2.0$, $\lambda = 2.0$, $\alpha = 10.0$, $\beta = 10.0$.

sulted in a final decision on the combination: $\gamma = 1.5$, $\lambda = 2$, $\alpha = B = 30$ (Note that other combinations were attempted, but results are not discussed here.) This final selection of weights is only optimal for this study. For other studies/functionals, the weight selection should be independently determined.

The final result for July 1984 (Fig. 9) is as desired, very comparable to the subjective analysis (Fig. 1). The difference field indicates some of the major discrepancies are spatially coherent. Some of these larger errors can be explained. Off the coast of Somalia, the objective 1-deg results were able to resolve a weakening of the Somalia jet whereas in the subjective analysis, originally on a 2-deg by 5-deg grid, these observations were averaged in with the larger values, and thus were not resolved. Increased resolution also accounts for many of the other discrepancies near coastal boundaries—the Indian peninsula, east coast of Africa, and Madagascar. Increased resolution was also the reason for the obviously spurious westerly at the eastern boundary, 11°N. In the southern trades, 70°–80°E, the weaker trades in the objective analysis were present regardless of coefficient selection. This is also the case at 100°E, where the objective analysis had weaker and more southerly trades. These two areas of difference are due to the subjective choice in the handdrawn analysis.

Aside from these few regions where there are major differences, the overall result is very good and compares favorably to the subjective analysis. The speed of the Findlater jet is nearly 15 m s^{-1}—very close to climatology. Cross-equatorial flow is also nearly normal near the African coast, but is higher than average from 55° to 70°E. The SCS winds are nearly normal, and the southern trades are anomalously strong. The curl maps of subjective and objective results compare very well in structure, but the subjective results give a noisier result. This is interesting since the subjective analysis is based originally on a 5° long × 2° lat grid, which is coarser than the objective results.

c. January 1984 study

The January 1984 analysis presents a contrasting test for optimal weight selection. For this study, the combination of weights was selected in similar fashion to the previous study of July winds. The quantitative results (Table 2) indicate the selection of weights is not a critical step in deriving the best quantitative results,

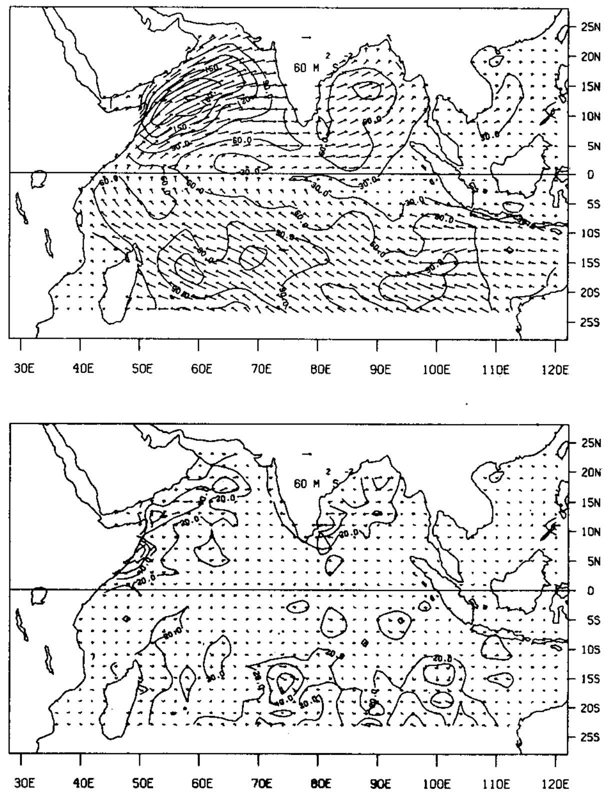

FIG. 9. (a) Pseudostress vector results of the variational (objective) analysis binned onto a 2° grid for July 1984. Contour interval is 30 $m^2 s^{-2}$. (b) Vector difference (objective results − subjective results). Contours are the vector difference magnitudes, and interval is 20 $m^2 s^{-2}$. This is the case for $\gamma = 1.5$, $\lambda = 2.0$, $\alpha = 30.0$, $\beta = 30.0$.

TABLE 2. Error characteristics for the difference field (objective results minus subjective results) for the January 1984 case.

Weights				$\lvert \mathbf{V}_D \rvert$	Kinetic energy
γ	λ	α	β	($m^{-2}\,s^{-2}$)	($\times 10^4\,m^{-4}\,s^{-4}$)
1	1	1	1	15.3	28.0
0.1	1	1	1	16.0	30.0
1.5	2	10	40	14.0	23.0
1.5	2	40	10	14.8	27.0
1.5	2	30	30	14.1	24.0

since some have comparable error levels. However, the resulting fields reveal that the selection does influence the distribution and characteristics of the difference field; objective–subjective. Similar to the difficulty in the July study, a certain weight selection can result in the nearest approximation to the independent, subjective analysis, but how that difference is distributed may not be satisfactory. Selecting weights to reduce a large difference at a particular grid point may induce larger differences at other locations. Thus quantitative error estimates in this research may not be sufficient enough to determine the best weight selection.

For this January study, the best weight selection was chosen to be the same ($\gamma = 1.5$, $\lambda = 2.0$, $\alpha = B = 30.0$) as in the July study (Fig. 10). Other weight selection results were similar in error level, but not in distribution.

The objective results compare very well to the subjective results, but again, the higher resolution of the objective analysis is responsible for the large differences along the Somalia coast, near Sri Lanka, and at the southern boundary. Southwest of Sumatra, at 8°S, 95°E, the objective analysis fails to allow a maxima of westerlies, which appears in the initial guess field. In the subjective analysis at 17°S, 70°E, the maxima is based on a single questionable wind report. In the SCS, the objective analysis fails to include a maxima in the southwest region (in fact this whole area is anemic) due to the subjective analysis of strong winds and on a weight selection that tended to minimize this large anomaly from climatology. (Other weight selections reduced this problem but created larger errors in other areas.) The curl fields—subjective and objective—again agree in sign and large-scale structure, but the objective results are smoother.

4. Conclusions

The technique of minimization has been demonstrated to be a viable addition to the collection of other objective techniques whereby pseudo-randomly distributed meteorological observations are used to fill a regularly spaced grid. Although in this paper the cost functional was limited to five terms, in practice, there is no limit on the number or structure of terms that can be included. The conjugate-gradient technique was very efficient in minimizing the functional. Significant speedup can be obtained by vectorizing the function and gradient calculation associated with the conjugate-gradient minimization of the cost functional (See Navon et al. 1988). A good initial guess field also aids in the rapid convergence of the technique.

Inherent in all objective analysis schemes are the tuning parameters. In our particular technique, the cost functional is itself a tuning parameter (what terms are included affects the results). More importantly, the weights of each of the terms must be selected either objectively (i.e., cross-validation or some variation) or subjectively, which involves quantitative and/or qualitative comparisons to independent analyses. Utilizing an independent subjective analysis of the same raw data in this research was the only option available, and while its resolution was coarser, it did provide guidance on how to best select the weights. The technique is essentially a filter with a response function being a function of functional design, climatology, and weight selection. The final choice of weights is reflected in the scales resolved in the objective analysis. The weights on the climatology and smoothing terms determined the overall magnitude of the results. The smoothing

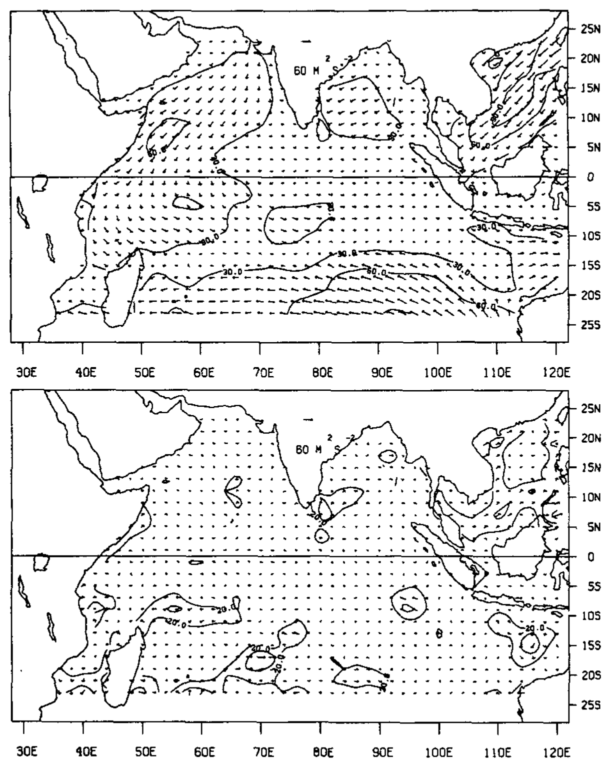

FIG. 10. (Top) Pseudostress vector results of the variational (objective) analysis binned onto a 2° grid for January 1984. Contour interval is 30 $m^2\,s^{-2}$. (Bottom) Vector difference (objective results − subjective results). Contours are the vector difference magnitudes, and interval is 20 $m^2\,s^{-2}$. This is the case for $\gamma = 1.5$, $\lambda = 2.0$, $\alpha = 30.0$, $\beta = 30.0$.

weight also determined the radius of influence of differences to climatology. The kinematic terms "fine-tuned" the results using climatology as a basis for desired physical qualities.

The results of this research have been used as the forcing term of a nonlinear reduced-gravity model of the Indian Ocean (Luther and O'Brien 1985). Results of a multiyear run of the model have provided additional verification (Simmons et al. 1988) of the wind products, and the model.

For future work, results from 1979 will be compared to FGGE level III-B data. Other topics to research include a better handling of boundaries, and how to possibly select the weights objectively. Are the proper weights a function of time of year, or perhaps a function of space? In addition, placing more weight on locations where there are numerous observations may help.

5. Summary

A technique not previously used in objective analysis of meteorological data is used to produce monthly average surface pseudostress data over the Indian Ocean. An initial guess field is derived from binning available merchant ship wind reports into 1-deg boxes and interpolating to fill in data voids. A cost functional was constructed which has five terms: approximation to initial guess; approximation to climatology; a smoothness parameter; and two kinematic terms, approximation of the divergence and curl of the resulting fields to the divergence and curl of the climatology. Each term had a weight associated with it. This functional was minimized using a conjugate-gradient technique. It was found that the weight for the climatology term controls the overall balance of influence between the climatology and the initial guess. The smoothing weight determined how smooth the result would be (all terms can smooth the results), by regulating how wide of an area the differences to climatology would influence. The kinematic weights are "fine-tuning" parameters which insured the results contained some level of climatic forcing. The weights were selected by quantitative and qualitative comparisons of results using various selections of weights to results of an independent subjective analysis of the same merchant ship data. The best selection of weights as determined by the quantitative comparison to the subjective analyses was not always the best qualitatively. Picking the weights to reduce large very localized differences often increased the difference field in other, widespread, regions. The results for two months, January and July 1984, indicate that while the same weight selection for both months provided satisfactory results, the weights may be a function of time of year or season.

Results of this research, the 1-deg gridded pseudostress values, are available to the atmospheric and oceanographic community in digital and graphical form. For more information, please contact the authors.

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APPENDIX

The conjugate-gradient algorithm, CONMIN is a Beale-restarted memoryless variable metric algorithm documented in Shanno (1978a, 1978b), Shanno and Phua (1980), and Navon and Legler (1987).

The method requires approximately 7N single/double precision words of working storage. The different steps of the implementation of CONMIN are the following:

(i) Initialization

Choose $X_0 = (U_{11} \cdots U_{N_xN_y}, V_{11} \cdots V_{N_xN_y})^T$ or $(\bar{\tau}_{x_{11}} \cdots \bar{\tau}_{x_{N_xN_y}}, \bar{\tau}_{y_{11}} \cdots \bar{\tau}_{y_{N_xN_y}})^T$. Set initial guess of Hessian $H_0 = \mathbf{I}$ (i.e. the unit matrix). Choose an accuracy parameter ϵ and get $k = 0$. Compute

$$F_k = F(\mathbf{X}_k)$$

$$\mathbf{g}_k = g(\mathbf{X}_k), \quad \text{i.e. the gradient of functional } F$$

$$\mathbf{s}_k = -\mathbf{g}_k \quad \text{and} \quad s_k{}^T g_k. \qquad \text{(A1)}$$

(ii) The linear search procedure for the step-size

In this step, an inexact linear search proposed by Shanno and Phua (1976) is implemented with some modifications.

The basic linear search implements a Davidon cubic interpolation to find an "optimal" step size α_k which satisfies the following two conditions:

$$F(\mathbf{X}_k + \alpha_k \mathbf{s}_k) < F(\mathbf{X}_k) + 0.0001\alpha_k \mathbf{s}_k{}^T \mathbf{g}_k \qquad \text{(A2)}$$

$$|s_k{}^T g(X_k + \alpha_k s_k) s_k{}^T g_k| < 0.9. \qquad \text{(A3)}$$

(iii) Test for convergence

Get $\mathbf{X}_{k+1} = \mathbf{X}_k + \alpha_k s_k$

$$F_{k+1} = F(\mathbf{X}_{k+1})$$

$$\mathbf{g}_{k+1} = g(\mathbf{X}_{k+1})$$

$$\mathbf{p}_k = \mathbf{X}_{k+1} - \mathbf{X}_k$$

$$\mathbf{y}_k = \mathbf{g}_{k+1} - \mathbf{g}_k. \qquad \text{(A4)}$$

If $\|g_{k+1}\| \leq \epsilon \max(1, \|X_{k+1}\|)$ stop. Else proceed to step (iv).

(iv) Perform the Beale restart according to Powell (1977) criterion

If the Powell (1977) criterion holds, perform a Beale (1972) restart described in this procedure. Otherwise proceed to step (v). Powell's restart criteria are the following:

(a) The conjugate-gradient iteration k is a multiple of n

(b) $|g_{k+1}^T g_k| \geq |g_{k+1}\|^2$. (A5)

If either of these two conditions holds compute a new search direction s_{k+1} by

$$s_{k+1} = \gamma g_{k+1} - \left[-1 + \frac{\gamma y_k^T y_k}{p_k^T y_k}\frac{p_k^T g_{k+1}}{p_k^T y_k} - \frac{\gamma y_k^T g_{k+1}}{p_k^T y_k}\right] p_k + \frac{\gamma p_k^T g_{k+1}}{p_k^T y_k} y_k \quad \text{(A6)}$$

where

$$\gamma = \frac{p_k^T y_k}{\mathbf{y}_k^T \mathbf{y}_k}. \quad \text{(A7)}$$

Set $p_t = s_k$, $y_t = \mathbf{y}_k$ and go to step (ii). Otherwise

(v) Compute a new search direction by the two-step memoryless BFGS formula (Shanno 1978)

$$s_{k+1} = -\hat{H}_k \mathbf{g}_{k+1} + \frac{\mathbf{p}_k^T \mathbf{g}_{k+1}}{\mathbf{p}_k^T \mathbf{y}_k} \hat{H} y_k - \left(1 + \frac{y_k^T \hat{H}_k y_k p_k^T g_{k+1}}{p_k^T y_k p_k^T y_k} - \frac{y_k^T \hat{H}_k g_{k+1}}{p_k^T y_k}\right) p_k. \quad \text{(A8)}$$

Here $\hat{H}_k$ is an approximation of the inverse Hessian using rank-two updates of the initial $H_0 = \mathbf{I}$ while the vectors $\hat{\mathbf{H}}_\mathbf{k}\mathbf{g}_{\mathbf{k}+1}$ and $\hat{\mathbf{H}}_\mathbf{k}\mathbf{y}_\mathbf{k}$ are defined by

$$\hat{\mathbf{H}}_\mathbf{k}\mathbf{g}_{\mathbf{k+1}} = \frac{p_t^T \mathbf{y}_t}{y_t^T y_t} g_{k+1} - \frac{p_t^T g_{k+1}}{y_t^T y_t} y_t + \left(\frac{p_k^T g_{k+1}}{p_t^T y_t} - \frac{y_t^T g_{k+1}}{y_t^T y_t}\right) p_t \quad \text{(A9)}$$

$$\hat{\mathbf{H}}_\mathbf{k}\mathbf{y}_\mathbf{k} = \frac{p_t^T \mathbf{y}_t}{\mathbf{y}_t^T \mathbf{y}_t} y_k \frac{p_t^T \mathbf{y}_k}{\mathbf{y}_t^T \mathbf{y}_t} y_t + \left(\frac{2 p_t^T y_k}{p_t^T \mathbf{y}_t} - \frac{\mathbf{y}_t \mathbf{y}_k}{\mathbf{y}_t^T \mathbf{y}_t}\right) p_t. \quad \text{(A10)}$$

As suggested by Fletcher (1972) the search vector is scaled by

$$s_{k+1} = 2(F_{k+1} - F_k)/g_{k+1}^T s_{k+1}. \quad \text{(A11)}$$

(vi) Storage requirements of CONMIN

From the description of the CONMIN algorithm (see also Shanno and Phua 1980), it is evident that the implementation of this algorithm would require the storage of the following vectors:

Vector	Description
$\mathbf{x}$	The current estimate of the minimum
$\mathbf{g}$	The gradient evaluated at the current point
$\mathbf{s}$	The current search direction
$\mathbf{x}^*$	The new estimate of the minimum
$\mathbf{g}^*$	The gradient evaluated at $x = x^*$
$\mathbf{s}_t$	The Beale restart search direction
$\mathbf{y}_t$	The Beale restart vector

Notice that no extra storage is required to store the vector y, since this vector can be stored into the vector $\mathbf{x}^*$ after the vector $\mathbf{x}$ is replaced by $\mathbf{x}^*$.

Consequently, the CONMIN subroutine requires $7n$ single/double precision real words of storage, in addition to the storage of various auxiliary scalar products.

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