The Measurement Method of the *Almagest* Stars

Dennis Duke, Florida State University

I suggest that the correct standard model of early Greek stellar astronomy is

A. Someone, perhaps Hipparchus, measured a fairly complete star catalog in *equatorial* coordinates.
B. That catalog was the basis for the results presented in Hipparchus' *Commentary to Aratus*.[1]
C. Analog computation was used to convert most of the catalog to *ecliptical* coordinates.
D. It is this converted catalog, with longitudes shifted by 2°40′, that we have received through Ptolemy and the *Almagest*.

The supporting argument in brief is:

The star coordinates in Hipparchus' *Commentary to Aratus* are clearly equatorial right ascensions and declinations.[2] Although we have no surviving hint *how* those coordinates were measured, or even *who* measured them, it is reasonable to assume that the coordinates were measured in the same way they were presented: equatorial coordinates. Ecliptical stellar coordinates are conspicuous in their absence.

The correlations between the errors in the *Almagest* data and the *Commentary* data show that those two data sets are associated in some way. This is substantiated by[3]

(a) several stars with large common errors in each data set,
(b) detailed statistical analysis of the error correlations between the two sets of data, and
(c) similar systematic errors in the two data sets.

These facts are most easily reconciled by assuming a catalog in equatorial coordinates that was used to calculate the *Commentary* data, and was eventually used in substantial part for the *Almagest* catalog. Strictly speaking, this catalog need not originate from Hipparchus.

The star coordinates in the *Almagest* are ecliptical longitudes and latitudes, which are clearly the most convenient form for any astronomer in the era Hipparchus – Ptolemy, whose primary interest would likely be lunar and planetary positions. Ptolemy claims that he measured the star coordinates with a zodiacal armillary sphere, but several analyses show that his claim must be largely not true, and that he must have copied or derived most if not all the coordinates from some other catalog,[4] adjusting the longitudes to account for precession.

We now invoke Newton's fractional ending observation to conclude that the catalog that Ptolemy copied *from* was, at the time he did the copying, also in ecliptical coordinates, but with excesses of 00′ endings in both longitude and latitude (excepting the southern constellation stars – see below). This implies that the catalog Ptolemy copied from was either the result of

(a) direct measurements in ecliptical coordinates, or
(b) conversion from equatorial coordinates by some method that resulted in excesses at 00′ endings in longitude and latitude.

However, direct measurement of the ecliptical position of each star would give coordinate errors that were statistically uncorrelated with the equatorial coordinate errors mentioned above, and so is hard to reconcile with the clear and strong common heritage of the *Commentary* and the *Almagest* data sets. This suggests, therefore, that the most likely scenario is that someone converted the original equatorial coordinates to ecliptical using some form of analog computation (again excepting the southern stars). Hipparchus using a celestial globe is an obvious candidate[5].

Supporting Discussion

In Book 2 of his *Commentary* Hipparchus gives us a wealth of data on the rising and setting of constellations. In particular, for each constellation he tells us the first and last star to rise and set, and the degree of the ecliptic on the horizon and the meridian at each such rising and setting. In addition, he mentions stars that are on or near the meridian at the moment of the rising or setting. Some authors have interpreted these passages to mean that Hipparchus was using the so-called polar longitudes, or *mediato coeli*, as actual star coordinates.[6] However, the conclusion that Hipparchus consistently used equatorial coordinates, and *only* equatorial coordinates, is based on the following observations:[7]

(a) in the *Commentary* Hipparchus actually quotes the positions of numerous stars directly in right ascension or declination (or more often its complement, polar distance),

(b) Polar longitudes are not directly measurable, since the measurement of any longitude is always with respect to some other previously measured longitude, and there is no way to measure one polar longitude with respect to another polar longitude.

(c) polar longitudes are in fact *never* quoted directly for a single star in the *Commentary,* and

(d) since Hipparchus did not measure the rising, setting, and culmination numbers directly in the sky, he must have *computed* the numbers somehow, using some *other* set of numbers as input to the calculation. Hipparchus gives an explicit example, and that example uses right ascension and declination as the initial input data.

The statistical evidence that the rising/setting phenomena data in the *Commentary* and the *Almagest* coordinates share a common heritage is substantial.[8] Figure 1 shows cases of stars with large and similar errors in both data sets. It is unlikely that independent observations of all these stars would result in essentially the *same* large errors. Omitting the outlier cases and analyzing the correlations between the smaller errors in the

Commentary and the *Almagest* also shows that the data sets most likely have a common heritage. The correlations are quantitatively understood by means of a simple model: the Almagest errors are ε_i, where ε has mean zero and variance σ_A^2, while the Commentary errors are $\varepsilon_i + \eta_i$, and these have mean zero and variance σ_C^2. The ε and η errors are completely uncorrelated, while the added η errors account for the empirical fact that the variance σ_C^2 in the Commentary errors is larger than the variance σ_A^2 in the Almagest errors. A simple extension of the model allows an estimate of the fraction of stars copied by Ptolemy and concludes that the fraction is large, and not inconsistent with unity. Finally, it is possible to estimate the *systematic* errors in the *Commentary* phenomena data, and they show a clear similarity to the systematic errors observed in the *Almagest* coordinates. Although the comparison of the *Almagest* and *Commentary* statistical errors is limited to the 134 stars common to both catalogs, the clear association between the *systematic* errors implies that the association is more broadly based, since the systematic errors are relatively smooth, few-parameter, collective effects that permeate the entire data sets in both the *Commentary* and the *Almagest* catalogs. These observations taken together thus strongly suggest that the *Commentary* data and the *Almagest* coordinates share, at least in large part, a common heritage. In the case of the *Commentary* we also know, as discussed above, that the heritage comes from a catalog expressed in equatorial coordinates.

Are the positions of the stars included in the *Almagest* catalog consistent with measurement with an armillary? Comparing the number of stars catalogued with the number easily seen in the sky (i.e. those with visual magnitude less than 5) reveals that the cataloger included 25 of the 28 stars (89%) within 15° of the ecliptic pole, and 12 of the 20 stars (60%) within 15° of the equatorial pole. Near the center he included 77 of 86 stars (90%) within 3° of the ecliptic, and 39 of 59 stars (66%) within 3° of the equator. Overall, he included 442 of the 730 stars (61%) north of the ecliptic and 444 of the 744 stars (60%) north of the equator, corresponding to a catalog limiting magnitude[9] of just under V = 5. Thus the star densities near the equator and its pole are consistent with the overall density of inclusion, while the densities near the ecliptic and its pole are substantially elevated.

However, when using an armillary sphere, either zodiacal or equatorial, it is particularly difficult to accurately measure stars near either the pole or the equator of the system[10]. The statistical error[11] distributions of the *Almagest* coordinates are shown in Figures 2-7, and they do not reveal any anomalous behaviors near either equator or pole. The fact that the star positions, especially the latitudes near the ecliptic or equator and the longitudes near either pole, are relatively well measured is hard to understand if the measurer used an armillary of any sort.

On the other hand, measuring the star positions in equatorial coordinates does not require an armillary. Indeed, one plausible scenario is that the declinations were determined by measuring the altitude (or zenith distance) of the stars at meridian transit, while the right ascensions could be determined by measuring the distance of the star from the standard star-clock star positions that Hipparchus noted in Book 3 of the *Commentary*.[12] Indeed, there is a much older (*ca.* 700 BC at the latest) Babylonian tradition of *ziqpu* star-clock observations,[13] so it would not be surprising that Hipparchus might have used a similar strategy. In any event, such measurement methods offer an essentially unobstructed view of the ecliptic, the equator, and both associated poles, and thus are much easier to reconcile with the selection of catalogued stars than the idea that an armillary sphere was used for the measurements.

The systematic errors in right ascension and zenith distance are shown in Figures 8 and 9, separated by the northern, zodiacal, and southern constellations as grouped by Ptolemy. I am showing here the systematic errors instead of the total errors because it is likely that the systematic errors, being much less contaminated with random noise, will most clearly reveal just *how* the star coordinates were measured. It is possible, of course, that some other grouping would reveal more interesting information. A comparison of the systematic errors in right ascension of the *Almagest* star positions with the errors in Hipparchus' star-clock positions is shown in Figure 8. If the declinations were determined by measuring the zenith distance z at meridian transit using the relationship $\delta = \varphi - z$, where φ is the geographical latitude of the observer, then it is possible that analysis of the data in Figure 9, perhaps along the lines suggested by Rawlins,[14] will yield interesting information.

Newton's analysis of the distribution of fractional endings suggests that someone added n°40′, with n an integer, to each ecliptical longitude. Thus if the original ecliptical longitude endings had excesses at 00′ then the *Almagest* longitude endings would have excesses at 40′, as Newton indeed observed to be the case. One option is that Ptolemy had a catalog that Hipparchus had himself converted to ecliptical coordinates. Another option is that someone did the conversion from equatorial to ecliptic at a later date, perhaps even Ptolemy himself. The sheer quantity of computation would be a good reason to resort to analog computation, no matter who did it. In any event, Ptolemy tells us directly that[15]

> "one has a ready means of identifying those stars which are described differently [by others]; this can be done immediately simply by comparing the recorded positions."

thereby implying that he was not the first to use ecliptical coordinates in a star catalogue.[16]

Table 1 gives the distribution of fractional endings for several groupings of stars. In preparing the table I have subtracted 2°40′ from the *Almagest* longitude for each star. If the original longitudes were binned like the latitudes, i.e. in bins of 00′, 10′, 15′, 20′, 30′, 40′, 45′, and 50′, then the subtraction will unfortunately not recover the original ending distributions, since the original cases of 15′ and 45′ cannot occur in the reverse process. This adds a layer of complexity to the analysis of each case. Newton suggested that Ptolemy rounded the 15′ + 40′ = 55′ cases to 00′ and the 45′ + 40′ = 25′ cases to 20′. If so, when we reverse the process the 00′ - 40′ = 20′ cases and the 20′ - 40′ = 40′ cases will show elevated populations, since some of their members should really be in the nearby 15′ and 45′ bins. This should be kept in mind when inspecting the distributions in Table 1.

On the other hand, under the scenario suggested in this paper the declinations and right ascensions were measured with two different instruments, and so it is not obvious that we should expect the same binning of observed values in each case. It is also possible, of course, that the equatorial to ecliptical conversion was a mixture of processes. The fact that the excess of 00′ and 30′ endings in latitude occurs for northern and zodiacal stars but not for southern stars,[17] whose endings are consistent with random distribution, was the basis for Rawlins' conclusion[18] that the southern stars were measured in equatorial coordinates by Hipparchus and then transformed to ecliptical using trigonometry. That may well be the case, and is worthy of further investigation. What is important, though, for the scenario suggested in this paper to be true, is that peaks at 00′ endings appear in longitude and latitude after the conversion process. This would definitely not be the case if the conversion was done exclusively using trigonometry, so it is essential that some form of analog conversion was used, at least for most of the catalog.

The scenario suggested in this paper differs from previous interpretations in various ways:

(1) Some previous interpretations of Hipparchus' catalog are that if he had one at all, it was expressed in a mixed system of non-orthogonal coordinates: declinations and polar longitudes.[19]

(2) Some authors thought that the analysis of Vogt[20] provided conclusive proof that the *Almagest* coordinates are original to Ptolemy, at least in large part.[21]

(3) Some authors have suggested that one way to understand the structure of the *Almagest* catalog is to assume certain reference stars were used to measure the ecliptical coordinates on a constellation-by-constellation basis.[22]

(4) Some authors have suggested that Hipparchus measured his catalog of star coordinates directly in ecliptical longitude and latitude, probably using a zodiacal

armillary sphere.[23] A partial exception, mentioned above, is the suggestion of Rawlins[24] that the southern stars were measured in equatorial coordinates.

Hopefully it will be fairly straightforward to find or cite additional evidence that either strengthens or refutes the presently proposed model. The following list of questions, while no doubt incomplete, represents issues that would likely benefit from additional thoughtful consideration:

(a) When, where, and how were the original equatorial measurements made? Also interesting, but perhaps hard to answer, is whether it was Hipparchus or someone else who made the measurements.

(b) Can one identify any *Almagest* catalog stars that were likely *not* measured in equatorial coordinates? How many independent sources of coordinates do we find in the *Almagest* catalog?

(c) What was the precision of the coordinates quoted in the original equatorial catalog? And related, how did the 10′ bin sizes in the *Almagest* arise?

(d) When and how was the transformation from equatorial to ecliptical coordinates accomplished?

Was a zodiacal armillary ever used by any ancient astronomer? The scenario suggested in this paper certainly does not require that either Hipparchus or Ptolemy ever used one for measuring star positions. It is quite possible, though, that one was used for measuring elongations near the zodiac between stars, planets, and the Moon. This would be relatively easy if the observer simply measured the difference in longitude between his target and a reference, assuming he had some way of knowing the longitude of the reference, e.g. using a table. We have numerous records of an Arabic tradition of the zodiacal armillary,[25] probably inspired by Ptolemy's description in the *Almagest*, but we have no surviving records of any substantial set of measurements made with one before the time of Ulugh Beg (ca. 1437). Applying the fractional ending test to Ulugh Beg's catalog seems to indicate that his data were measured in ecliptical coordinates,[26] but we also know that he had many other instruments to use, and we have little information about how any of his measurements were made. We know that Tycho Brahe built one but found it so difficult to use that he quickly abandoned using it.[27] It is possible that Ulugh Beg's catalog might provide a useful test case for further investigation of some of the issues raised in this paper.

Acknowledgements

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	Longitudes							Latitudes								
endings	00	10	20	30	40	50	N	00	10	15	20	30	40	45	50	N
All Stars																
North	96	45	62	61	68	29	361	110	30	34	38	76	38	10	25	361
Zodiac	94	24	82	52	58	35	345	66	31	28	33	81	50	21	40	350
South	51	32	79	65	58	32	317	50	44	33	40	53	40	17	40	317
total	241	101	223	178	184	96	1023	226	105	95	111	210	128	48	105	1028
Constellation stars																
North	87	41	54	56	66	28	332	101	24	32	35	72	34	9	25	332
Zodiac	83	23	63	39	50	29	287	60	25	22	23	65	38	20	37	290
South	51	30	70	62	55	30	298	47	42	31	38	50	37	16	37	298
total	221	94	187	157	171	87	917	208	91	85	96	187	109	45	99	920
informata stars																
North	9	4	8	5	2	1	29	9	6	2	3	4	4	1	0	29
Zodiac	11	1	19	13	8	6	58	6	6	6	10	16	12	1	3	60
South	0	2	9	3	3	2	19	3	2	2	2	3	3	1	3	19
total	20	7	36	21	13	9	106	18	14	10	15	23	19	3	6	108
bins of declination																
(45,90)	33	10	18	22	21	11	115	28	4	16	14	24	15	2	12	115
(15,45)	68	28	50	47	54	24	271	79	27	25	33	55	29	9	15	272
(-15,15)	66	21	74	38	58	33	290	56	33	26	26	64	41	15	33	294
(-45,-15)	70	41	73	64	47	27	322	61	35	26	32	66	40	19	43	322
(-90,-45)	4	1	8	7	4	1	25	2	6	2	6	1	3	3	2	25
total	241	101	223	178	184	96	1023	226	105	95	111	210	128	48	105	1028
bins of right ascension																
(0,90)	64	31	70	41	48	26	280	64	26	29	29	56	30	14	33	281
(90,180)	27	9	51	51	45	24	207	35	28	23	28	38	30	7	21	210
(180.270)	81	30	52	43	47	21	274	55	29	26	29	64	32	9	30	274
(270,360)	69	31	50	43	44	25	262	72	22	17	25	52	36	18	21	263
total	241	101	223	178	184	96	1023	226	105	95	111	210	128	48	105	1028

Table 1. The number of fractional endings in longitude and latitude for various groupings of stars. The longitudes have 2°40′ subtracted from their *Almagest* values. North, Zodiac, and South in the first column denote the groups of northern, zodiacal, and southern constellations as defined in the *Almagest*. The five *Almagest* stars with longitudes ending in 15′ are omitted from the table.

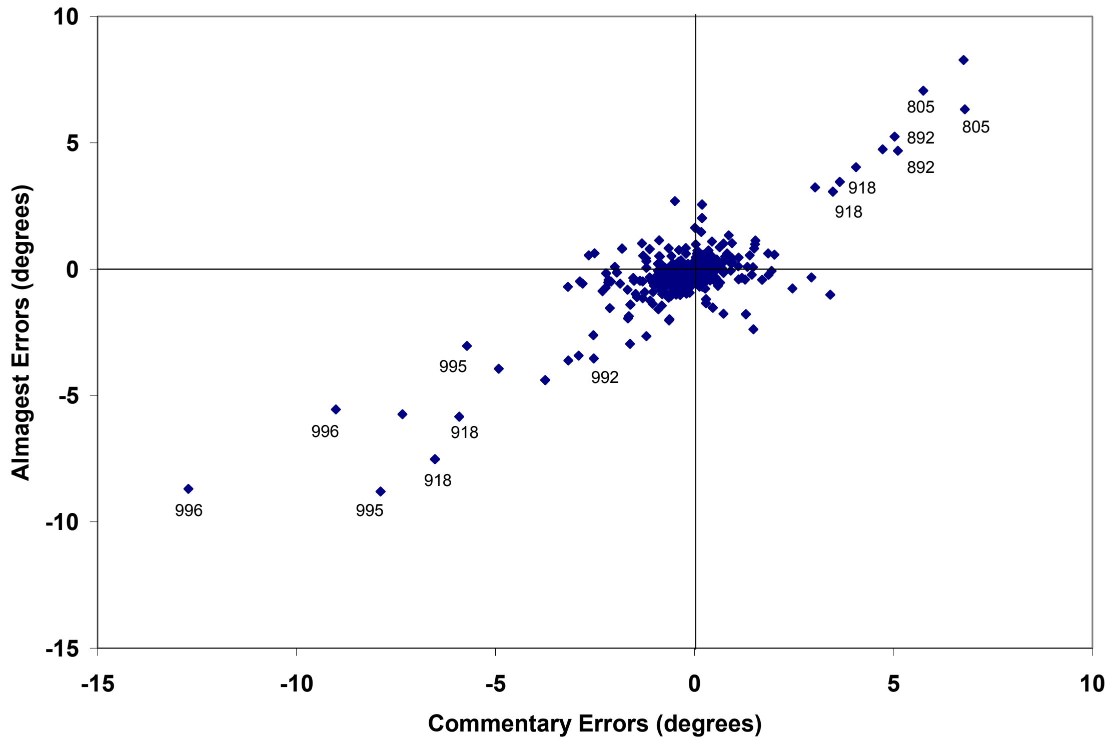


Figure 1. A scatter plot showing the correlation of the *Commentary* and *Almagest* errors for phenomena of types 1-4. Stars with large shared errors are marked with their Bailey number (the number of the star in the *Almagest* catalog).

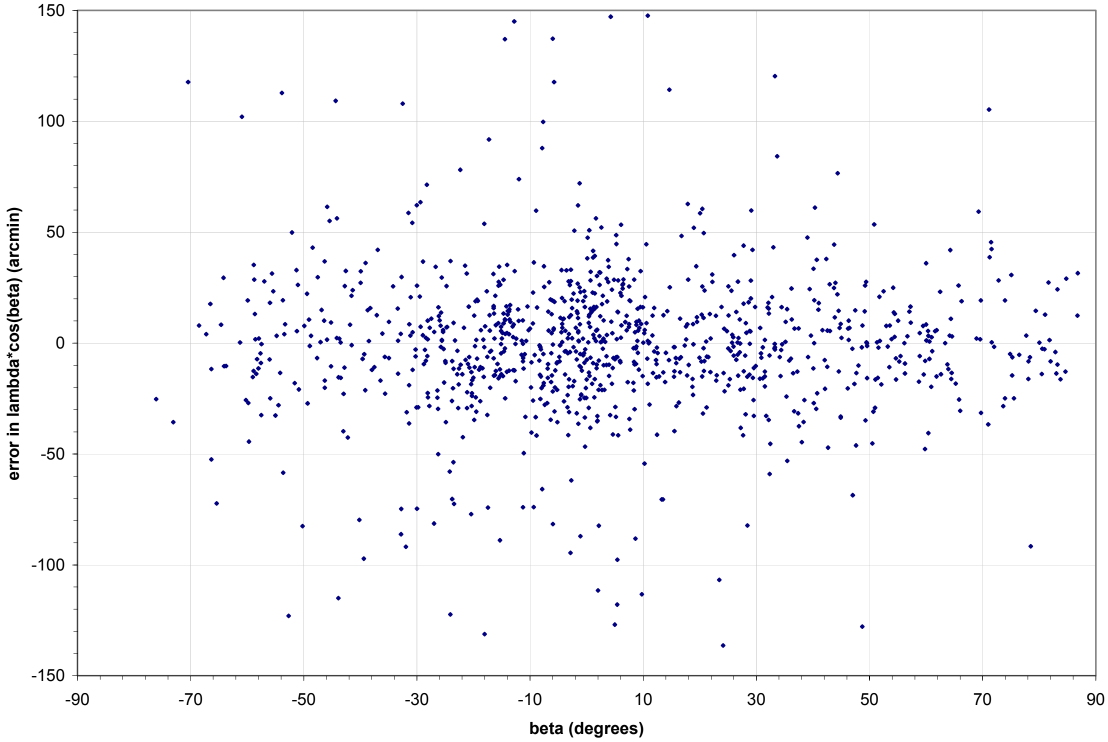


Figure 2. The statistical errors in longitude (reduced to the great circle) of the 1,028 *Almagest* stars.

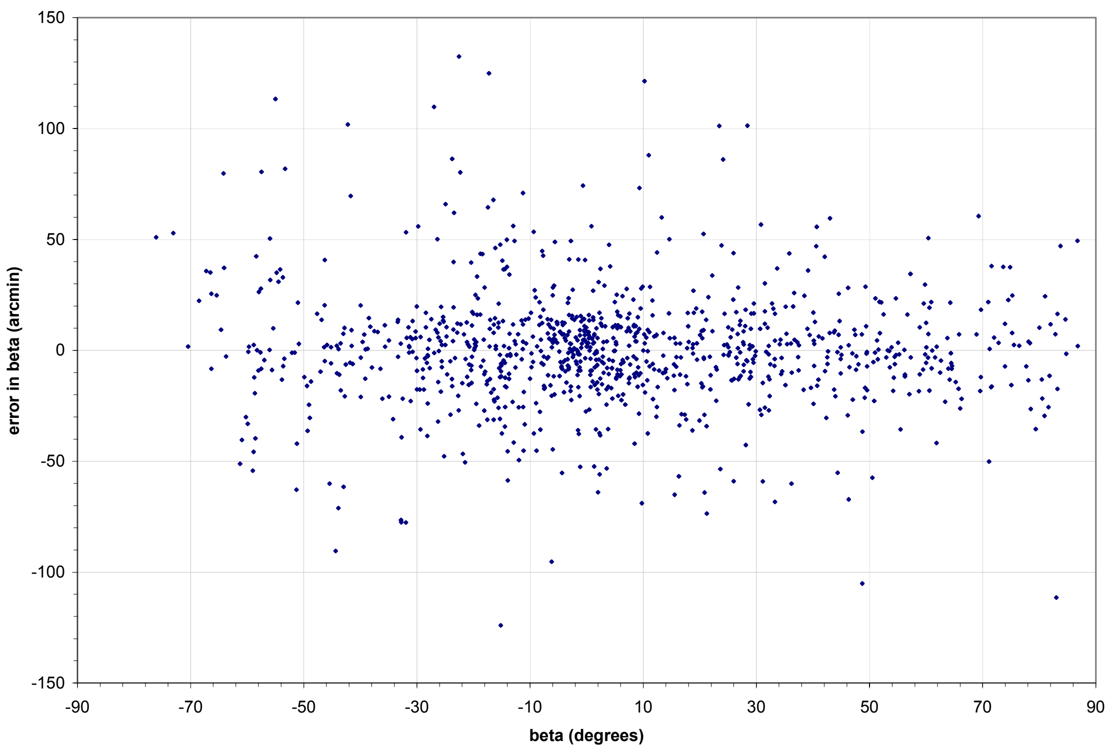


Figure 3. The statistical errors in latitude of the 1,028 *Almagest* stars.

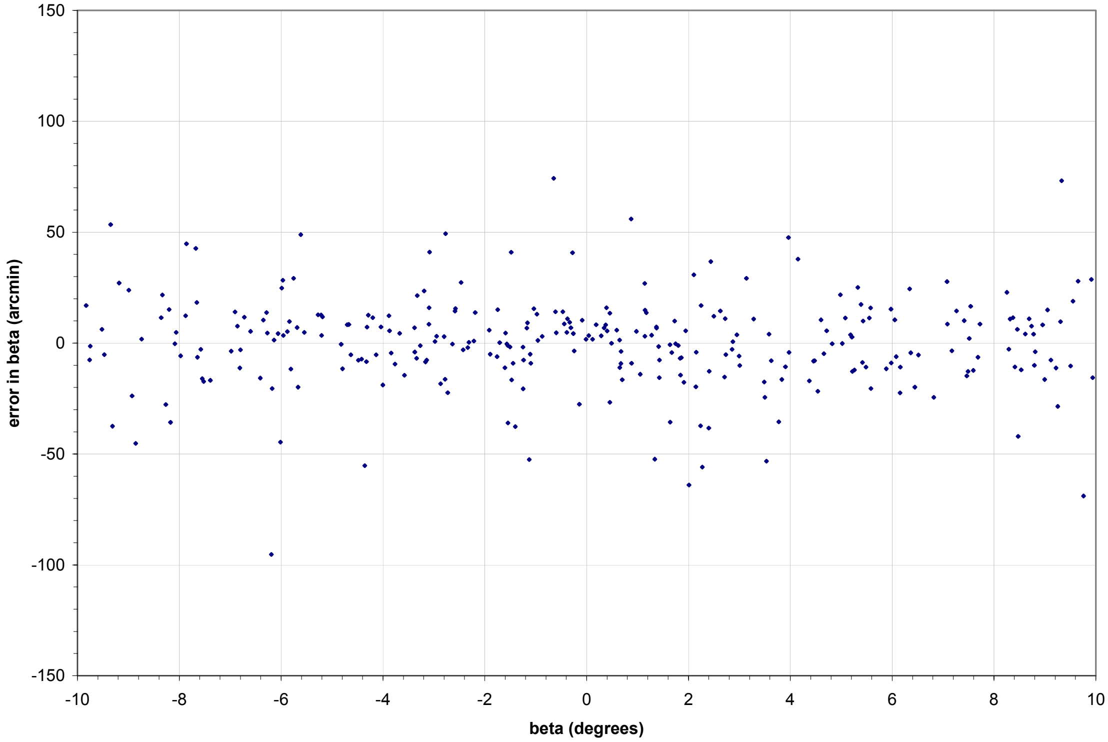


Figure 4. As in Figure 3 but looking close to the ecliptic.

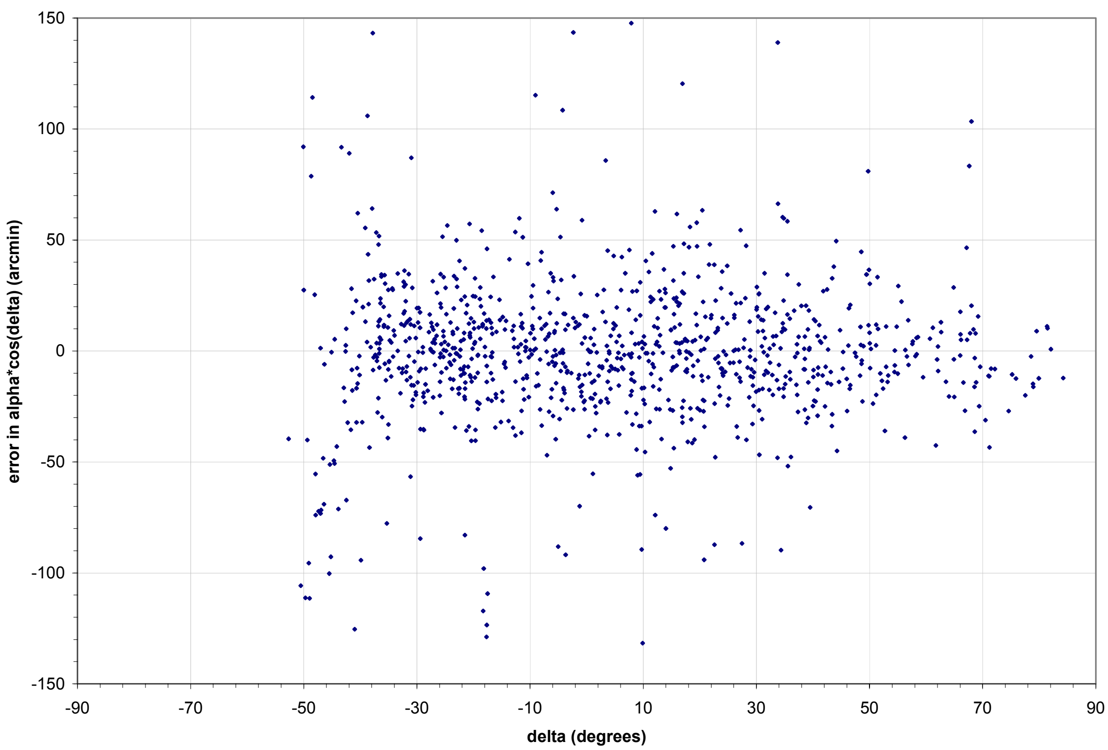


Figure 5. The statistical errors in right ascension (reduced to the great circle) of the 1,028 *Almagest* stars.

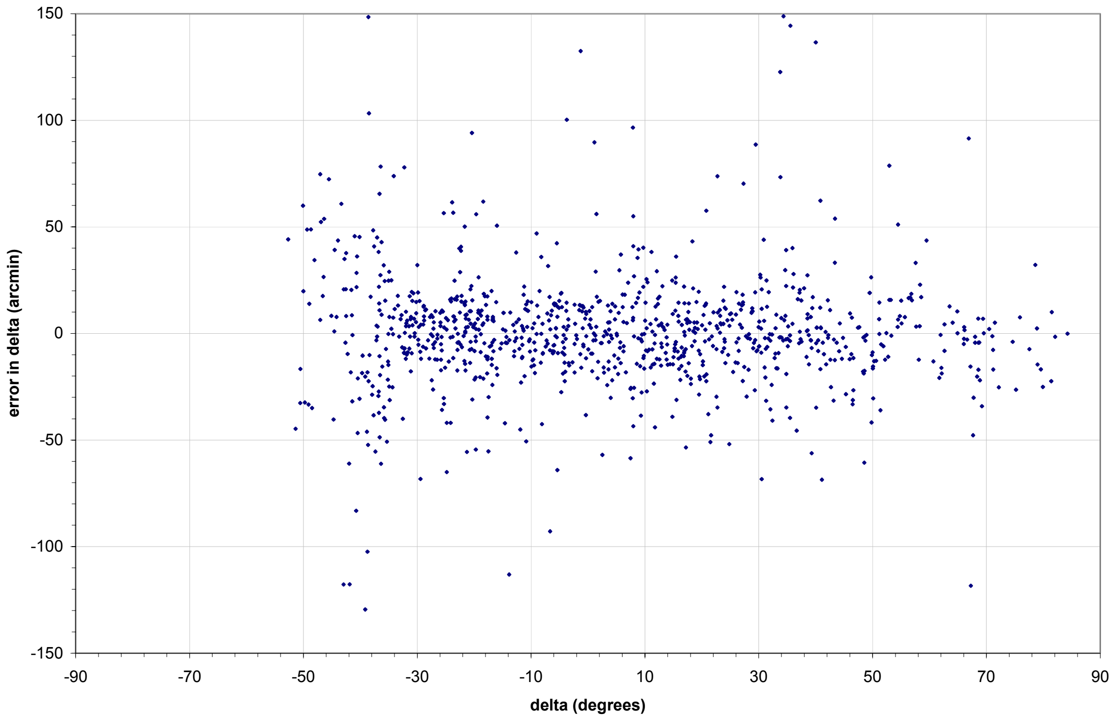


Figure 6. . The statistical errors in declination of the 1,028 *Almagest* stars.

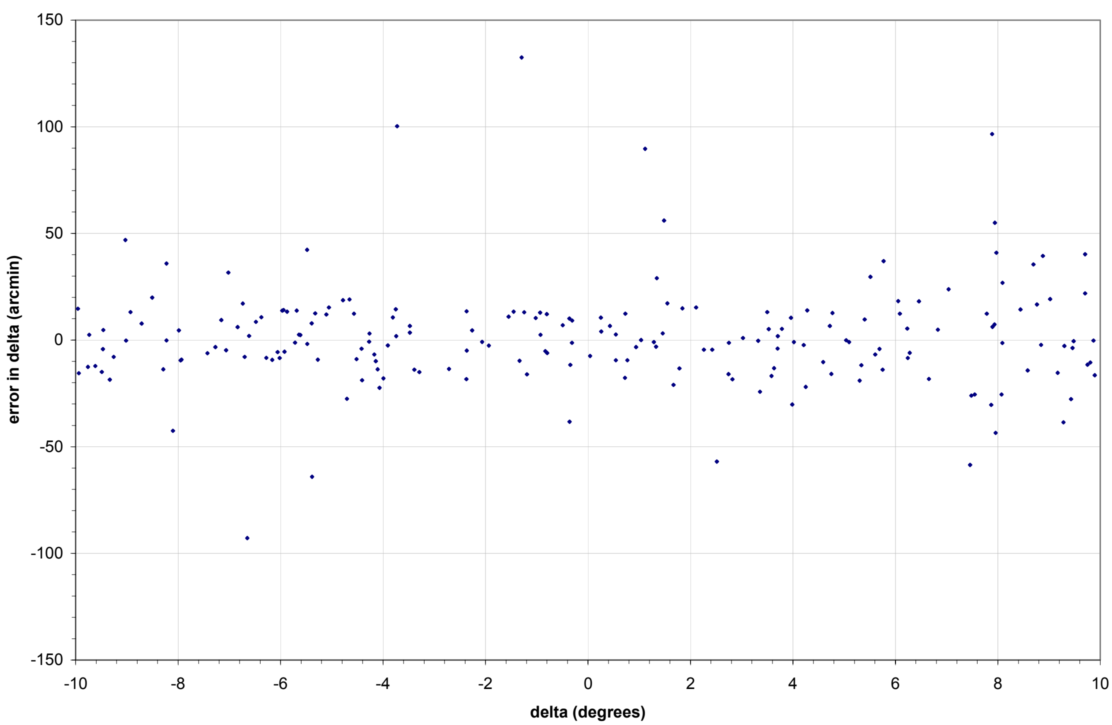
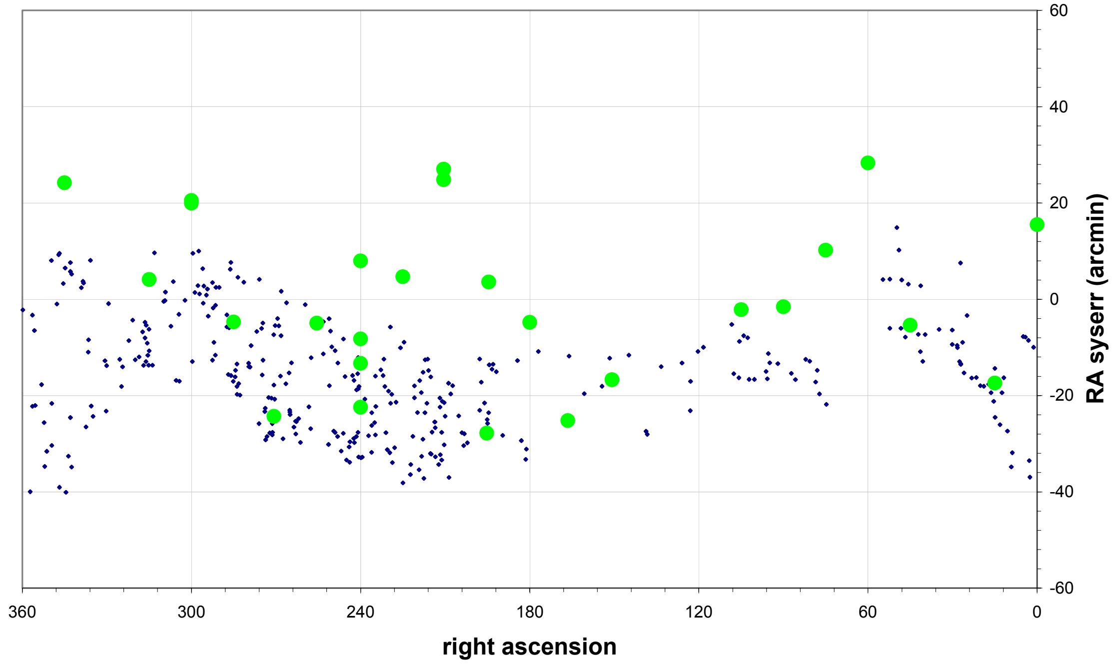
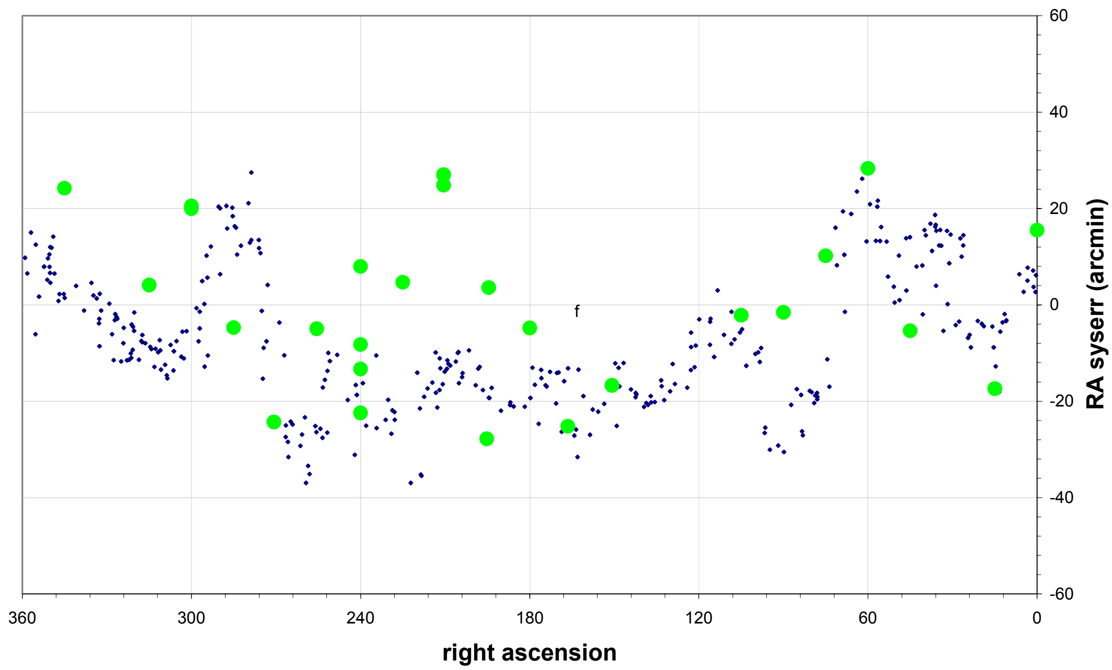


Figure 7. As in Figure 6 but looking close to the equator.

northern constellations



zodiacal constellations



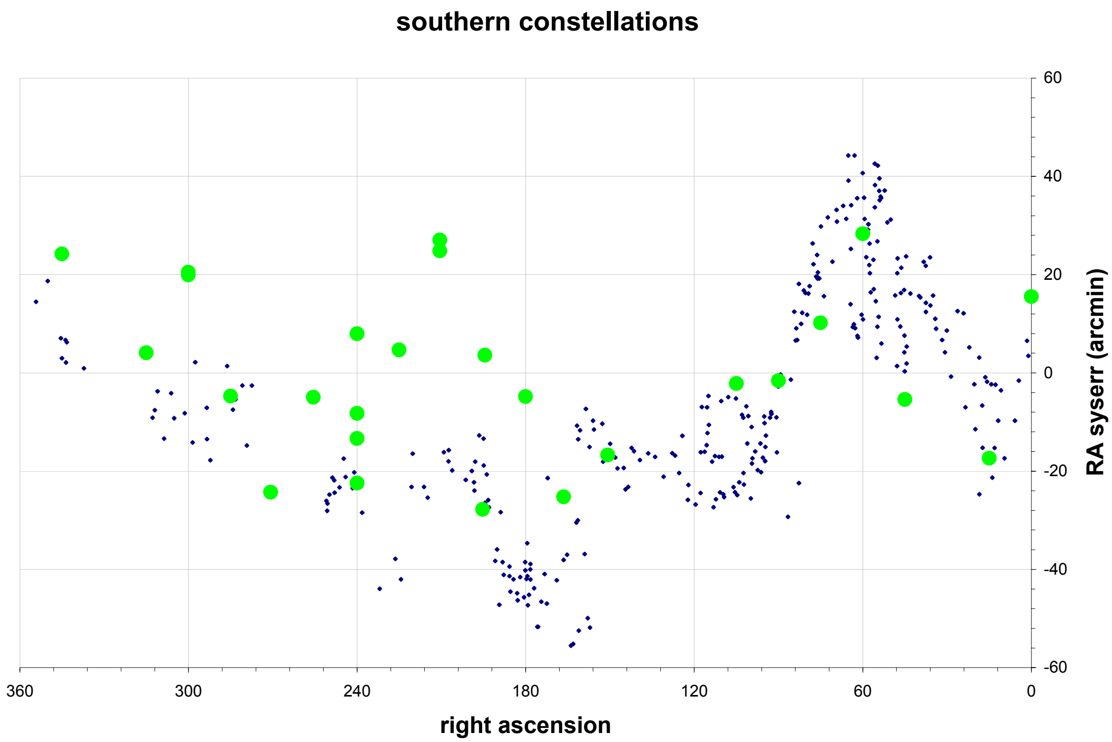


Figure 8. The systematic errors in right ascension, weighted by $\cos\delta$. The larger light circles are the errors in right ascension for the Hipparchan clock-stars with visual magnitude brighter than 4, which might have been used as reference stars to measure the right ascensions of target stars.

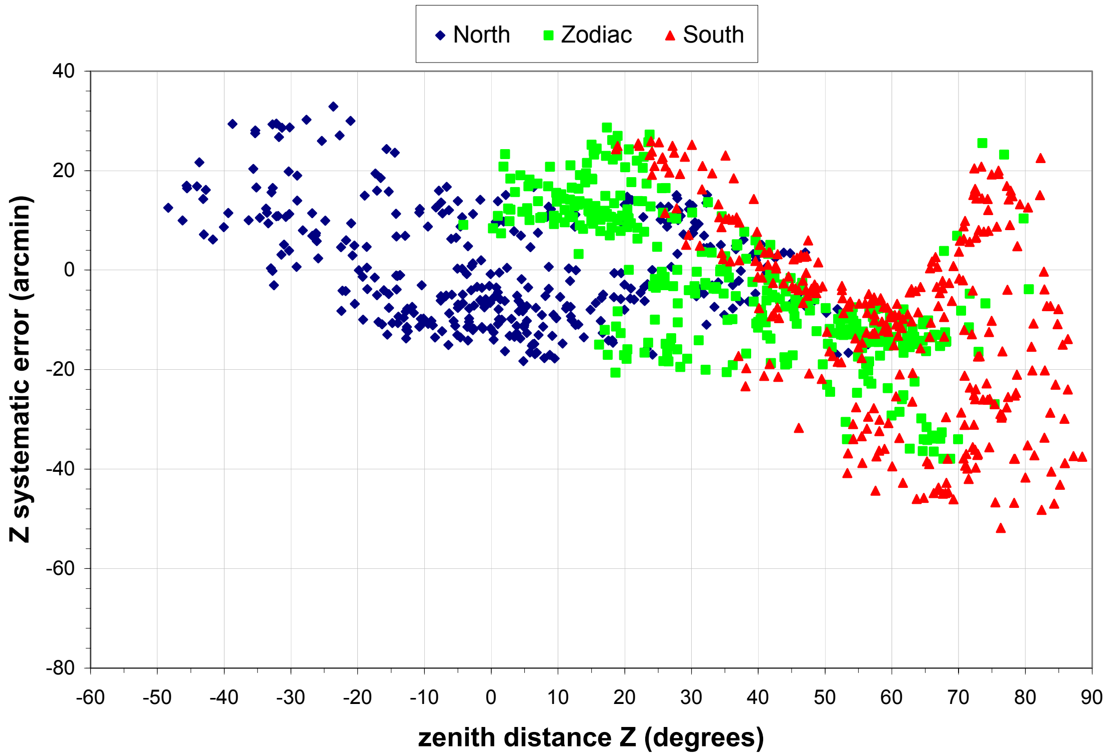


Figure 9. Systematic errors in zenith distance z, where z = φ – δ. The stars in the northern, zodiacal, and southern constellations are shown separately.

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[2] D. Duke, “Hipparchus’ Coordinate System”, *Archive of the History of the Exact Sciences* 56 (2002) 427-433.

[3] G. Grasshoff, *The history of Ptolemy’s star catalogue* (New York, 1990); D. Duke, “Associations between the ancient star catalogues”, *Archive for History of Exact Sciences,56 (2002) 435-450*; D. Duke, “The Depth of Association between the Ancient Star Catalogues”, *Journal for the History of Astronomy* (forthcoming).

[4] J. B. B. Delambre, *Histoire de l’astronomie ancienne* (2 vols, Paris, 1817), ii. 261-4; R. R. Newton, *The crime of Claudius Ptolemy*, (Baltimore, 1977); D. Rawlins, “An investigation of the ancient star catalog”, *Publications of the Astronomical Society of the Pacific*, xciv (1982), 359-73; *ibid.*, DIO 1.1 (1991), 62-63; *ibid.*, DIO 2.3 (1992) 102-113; G. Grasshoff, *op. cit.* (ref 3).

[5] R. Nadal and J.-P. Brunet, “Le Commentaire d’Hipparque I. La sphère mobile”, *Archive for history of exact sciences*, 29 (1984), 201-36 and “Le Commentaire d’Hipparque II. Position de 78 étoiles”, *Archive for history of exact sciences*, 40 (1989), 305-54. And, of course, Ptolemy tells us explicitly in *Almagest* VII.1 that Hipparchus had a globe.

[6] See, for example, O. Neugebauer, *A history of ancient mathematical astronomy*, (3 vols., Berlin, 1975), p. 277-80; G. J. Toomer, *Hipparchus*, Dictionary of Scientific Biography 15 (1978), p. 217; J. Evans, *The History and Practice of Ancient Astronomy*, (New York, 1998), p. 103; G. Grasshoff, “Normal star observations in late Babylonian astronomical diaries”, *Ancient astronomy and Celestial Divination* (1999), ed. N. Swerdlow, p 127 and footnote 23.

[7] D. Duke, *op. cit.* (ref. 2) gives complete details.

[8] D. Duke, *op. cit. (*ref. 3).

[9] D. Rawlins, *op. cit.* (ref. 4); B. Schaefer, "The latitude of the observer of the Almagest star catalogue, *Journal for the history of astronomy*, 32 (2001), 1-42.

[10] Primarily because the rings themselves obstruct the view of a star near either the pole or the equator of the instrument.

[11] I use the method of Dambis-Efremov to estimate these errors. See A. K. Dambis and Yu. N. Efremov, "Dating Ptolemy's star catalogue through proper motions: the Hipparchan epoch", *Journal for the history of astronomy*, 31 (2000), 115-134. See also D. Duke, "Dating the almagest star catalogue using proper motions: a reconsideration", *Journal for the history of astronomy*, 33 (2002) 45-55, which explains in a little more detail how to separate the statistical and systematic errors.

[12] One way that Hipparchus might have used is to construct a V-shaped instrument with two pieces of wood, perhaps a meter long, with a string across the top of the V, perhaps marked with equal increments of 1/15th the length of the chord. He would adjust the length of the chord so that the angle is 15°, something he definitely knew how to do.

Then, *assuming he has his star-clock table at hand*, he waits until a star transits, and keeping his instrument level, measures the distance, or number of 1/15th increments, to the nearest star-clock star. If the target star is near the equator, he is done. If it is at some non-negligible distance from the equator, he would have to correct for what we call the cos δ factor, but we know from the *Commentary* he knew how to do that, too. Using the chord as described is equivalent to linear interpolation in a table of chords, but he might have figured out how to do better. For all we know, some reasoning like this *led* him to the table of chords. How did he get the star-clock table? He only needs one star to start, then he can use the above procedure to bootstrap his way around the equator. Presumably he can get that one star from an observation during a lunar eclipse. Certainly his solar theory was adequate to get the accuracy we know he eventually published in the *Commentary* star-clock lists. Or perhaps he used the moon and his lunar theory, which was probably accurate enough near a full moon.

[13] J. Schaumberger, "Die ziqpu-Gestirne nach neuen Keilschrifttexten", *Zeitschrift fur Assyriologie,* 50 (1952) 214-229; B. L. van der Waerden, *Science Awakening II: the Birth of Astronomy* (1974) 77-79; D. Pingree and C. Walker, "A Babylonian Star-Catalogue:

BM 78161", *A Scientific Humanist: Studies in Memory of Abraham Sachs* (1988), 313-322; H. Hunger and D. Pingree, *Astral Sciences in Mesopotamia*, (Boston, 1999) 84-89.

[14] D. Rawlins, *DIO* 4.1 (1994) 33-47

[15] *Ptolemy's Almagest*, trans. by G. J. Toomer (London, 1984), p. 340.

[16] And further, since he says the comparison may be done 'immediately' and 'simply', Ptolemy is perhaps also telling us that other star catalogues in ecliptical coordinates were readily available, both for his readers and for himself (Noel Swerdlow, private communication, 2001).

[17] M. Shevchenko, An analysis of errors in the star catalogues of Ptolemy and Ulugh Beg", *Journal for the history of astronomy*, 21 (1990), 187-201.

[18] D. Rawlins, *op. cit.* (ref. 14).

[19] See ref. 6.

[20] H. Vogt, "Versuch einer Wiederstellung von Hipparchs Fixsternverzeichnis", *Astronomische Nachrichten*, 224 (1925), cols 2-54.

[21] O. Neugebauer, *op. cit.* (ref. 6), p. 280-4; G. J. Toomer, *op. cit.* (ref. 6), p. 217; N. M. Swerdlow, "The enigma of Ptolemy's catalogue of stars", *Journal for the history of astronomy*, 23 (1992), 173-183; J. Evans, "The Ptolemaic star catalogue", *Journal for the history of astronomy*, 23 (1992), 64-68.

[22] J. Evans, "On the origins of the Ptolemaic star catalogue", *Journal for the history of astronomy*, 18 (1987), 155-172, 233-278; M. Shevchenko, *op. cit.* (ref. 17); J. Wlodarczyk, "Notes on the compilation of Ptolemy's catalogue of stars", *Journal for the history of astronomy*, 21 (1990), 283-95.

[23] R. R. Newton, *op. cit.* (ref. 4) 255-6; D. Rawlins, *op. cit.* (ref. 14); K. Pickering, *DIO* 9.1 (1999), 26-29.

[24] D. Rawlins, *op. cit.* (ref. 14).

[25] Aydin Sayili, *The Observatory in Islam*, (1960).

[26] James Evans, *op. cit.* (ref. 21).

[27] Victor Thoren, *The Lord of Uraniborg: a Biography of Tycho Brahe*, (1991), p. 165.