Math 3040 Wrapup

M. M. Sussman

sussmanm@math.pitt.edu

Office Hours: 11:10AM-12:10PM, Thack 622

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Course topics

1. Background and basics
2. **`FEM1D.py`**
3. FEniCS
4. FreeFem++

Background and basics

- VirtualBox and VMs
- Spyder and Python language
- **`numpy`**, **`scipy.linalg`**, **`matplotlib.pyplot`**
- Basic FEM theory
  - Function spaces
  - Variational formulations (integration by parts)
  - Trial and test functions
  - Matrix and vector assembly by elemental summation

FEM1D.py

- Quadratic elements (for debugging)
- Class structure like FEniCS
- Direct solve using **`scipy.linalg.solve`**
- Gauß integration
- Testing and debugging

FEniCS

- Python interface
- UFL form language
- Linear algebra using PETSc or Trilinos
- Parallel capabilities
- Dimensional invariance
- Built-in direct and iterative linear solvers
- Built-in preconditioners
- Built-in algebraic and geometric multigrid solvers/preconditioners
- Newton's method, other solvers. Built-in or programmed
  - Automated differentiation for Jacobian construction
- General boundary conditions
- Eigenvalues/eigenvectors
- Transient applications, programmed
- NSE/vortex-shedding presented
- Mesh refinement
- Built-in optimization

FreeFem++

- C++ type interface language
- Coordinate-based form language
- Macros
- **`convect`** function
- Pressure penalty term in Navier-Stokes
- Array syntax and linear algebra
- Eigenvalues/eigenvectors
- Flexible mesh 2D generation using mapping functions
- Parallel implementation requires programming
- Sophisticated mesh adaptation
- Built-in optimization
- 3D mesh generation via **`TetGen`** or layers
- Many different applications presented

Advantages of each

FEniCS

- Dimensional invariance
- Parallel capabilities
- Parallel and serial direct solvers
- Iterative solvers and preconditioners
- Automated differentiation
- Nonlinear solvers
- Optimization

FreeFem++

- Compact, easy syntax
- `convect` function
- Serial direct and iterative solvers
- Macros
- Complex 2D mesh generation
- Sophisticated mesh adaptation
- Optimization

Summary of examples

FEniCS

1. 2D Poisson, Dirichlet b.c., exact solution
2. 1D Poisson
3. 3D Poisson
4. Gradient computations, projections
5. Convergence study
6. Multiple b.c.
7. 2-region Poisson
8. Parallel, iterative solution
9. Mesh generation, CGAL functions
10. Nonlinear heat conduction, Picard iteration
11. Nonlinear heat conduction, Newton iteration
12. Newton with automated Jacobian
13. Automated nonlinear solve
14. Transient heat conduction
15. Eigenvalues/eigenfunctions
16. Vortex-shedding
17. Vortex-shedding, Chorin's method

FreeFem++

18. Vortex-shedding
19. Poisson equation on circle
20. Elastic membrane, Dirichlet and Neumann
21. Elastic membrane, error
22. Heat exchanger
23. Acoustics: eigenvalues/eigenvectors
24. Bimetal plate
25. Nonlinear radiation b.c.
26. Airflow cooling hot blade
27. Rotating hill: `convect`
28. Rotating hill: DG
29. Rotating hill: efficient DG
30. Elasticity
31. Driven cavity: Stokes flow
32. Backward step, Chorin, adaptive mesh
33. Steady NSE, Newton, viscosity continuation, adaptive mesh
34. Schwarz method in parallel
35. Non-overlapping Schwarz

Summary of examples, cont'd

FreeFem++

36. Microwave oven
37. Optimal control: parameter ID
38. Shock wave, adaptive mesh
39. Mixed method, RT0 element
40. 3D mesh generation: "washer"
41. Cube mesh by layers
42. Cone mesh by layers
43. Adapting mesh on solution
44. Adapting mesh on gradient
45. Adapting mesh on indicator function